Unit 16 · Magnetic Fields

STUDY NOTES

WHAT NEB'S GRADE 12 EXAM ASKS

- **CDC's grid (2078):** Electricity and Magnetism (Units 14–19 together): 21 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** MCQ: NIA is a coil's **magnetic moment**; short (5): define flux density; a weak field (0.02–0.04 T) between two magnets on a steel yoke: a Hall probe ruled out, then measured with a current-carrying wire between the poles (the force $F = BIl$ for a known I and l gives $B = F/Il$)
- **MCQ (1 mark)**
  - turning a magnet through 180°: $W = 2MB$
  - axial field at x, $2x$: about 8 : 1
  - toroid with a core: $B = \mu_r\mu_0 NI/2\pi r$
  - magnetic moment ÷ angular momentum: $q/2m$
  - two crossed coils: $\sqrt{2}\times$
  - ¾ of a loop with straight leads
  - square frames: where B is not zero
  - galvanometer: $\theta = (NAB/k)I$
  - loop beside a wire: moves towards it
- **Short and long (5 or 8 marks, in parts)**
  - define flux, flux density; units; 1 T
  - Lorentz force (vector form), Fleming's left-hand rule; a charge's path in a field; speed unchanged; v for $2q$
  - force on a wire: $F = BIl\sin\theta$; balancing a rod's weight; a wire's p.d. to float
  - torque on a coil; galvanometer: principle, radial field, sensitivity
  - Hall effect: state, derive V_H, faces, why small in copper; a numerical
  - Biot–Savart: state; center of a coil (with figure); on the axis
  - Ampère's law; parallel wires attract (with figure); define the ampere; AC wires don't move
  - iron filings round a wire; field lines between two wires; where $B = 0$

Key terms

Magnetic flux ϕ: area × field component normal to it; scalar; weber (Wb).

Magnetic flux density B: flux per unit area at right angles to the field; vector; tesla (T); also the force on 1 C moving at 1 m/s at right angles to it.

Magnetic field intensity H: field made by currents alone; A/m (CGS oersted); in air $B = \mu_0 H$.

Lorentz magnetic force: $\vec{F} = q(\vec{v}\times\vec{B})$ on a moving charge.

Magnetic dipole moment: $M = NIA$ of a coil, along its normal; A m^2.

Current sensitivity: deflection per unit current, θ/I; **voltage sensitivity**: deflection per unit p.d., θ/V.

Hall effect: a voltage across a current-carrying conductor in a field at right angles to the current, at right angles to both.

One ampere: the steady current in each of two infinitely long, straight, parallel conductors of negligible cross-section 1 m apart in a vacuum that gives a force of **2×10^{-7} N per meter** between them.

Formulas

$\phi = BA\cos\theta$	θ from the **normal** **Use:** flux; $N\phi$ linkage
$F = Bqv\sin\theta$	θ between $\vec{v}$, $\vec{B}$ **Use:** force on a charge
$r = \dfrac{mv}{qB},\ T = \dfrac{2\pi m}{qB}$	$v \perp B$ **Use:** circle; T not on v
$F = BIl\sin\theta$	uniform field **Use:** force on a wire
$\tau = NBIA\cos\alpha = MB\sin\theta$	α plane–field, θ normal–field **Use:** torque on a coil
$I = \dfrac{k}{NBA}\theta$	k torsion constant **Use:** galvanometer
$V_H = \dfrac{BI}{net}$	t thickness along B **Use:** Hall voltage
$dB = \dfrac{\mu_0}{4\pi}\dfrac{I\,dl\sin\theta}{r^2}$	$\mu_0 = 4\pi\times10^{-7}$ H/m **Use:** Biot–Savart
$B = \dfrac{\mu_0 NI}{2r}$	N turns **Use:** center of a coil
$B = \dfrac{\mu_0 I}{4\pi a}(\sin\varphi_1 + \sin\varphi_2)$	long: $\mu_0 I/2\pi a$ **Use:** straight wire
$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enc}}$	steady currents **Use:** Ampère's law
$\dfrac{F}{l} = \dfrac{\mu_0 I_1 I_2}{2\pi r}$	like attract **Use:** parallel wires

Field and flux

- **Oersted** (1820): a current turns a compass, only while it flows; more current → more turn; reversed → other way
- lines of force: out of N, into S outside; S → N inside (**closed loops**); **never cross**; closer = stronger; side by side they push apart, along themselves they shorten
- 1 T = 1 Wb/m^2 = 1 N/(A m) = **10^4 G**; 1 Wb = 10^8 Mx = 1 T m^2 = 1 V s; $[\phi] = [ML^2T^{-2}A^{-1}]$
- tesla and gauss measure **B**; A/m and oersted measure **H**
- **neutral point**: magnet's field cancels the Earth's horizontal field (compass points nowhere); N pole pointing north → on the **equatorial line** (east, west); N pole pointing south → on the **axial line**

Direction rules

- **right-hand thumb rule** (straight wire): thumb along the current, fingers curl with the field
- **Maxwell's corkscrew rule**: screw moves along the current, handle turns with the field
- **right-hand grip rule** (coil, solenoid): fingers with the current, thumb along the field (N pole)
- **Fleming's left-hand rule**: First finger Field, seCond finger Current, thuMb Motion (force)

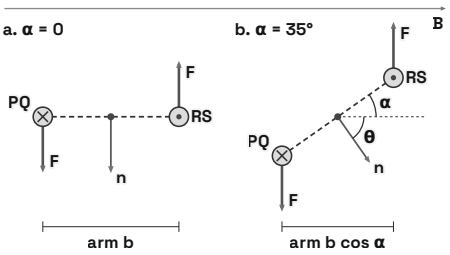
Moving charges

- $F \propto q$, $F \propto v \sin\theta$, $F \propto B \rightarrow F = Bqv\sin\theta$; defines the tesla
- $\theta = 90°$: largest, Bqv; along the field (0°, 180°): **0**; at rest: **0**
- force $\perp \vec{v} \rightarrow$ **does no work**; changes direction, not speed (KE constant)
- $\perp$ field: $Bqv = mv^2/r \rightarrow$ circle; same period for every speed (**cyclotron frequency** $qB/2\pi m$)
- at an angle: $v\cos\theta$ along B unchanged + circle of $v\sin\theta$ → **helix**, pitch $p = 2\pi mv\cos\theta/qB$
- same path with charge $2q$: $r = mv/qB \rightarrow$ speed $2v$
- electron: force the other way
- crossed E and B: $qE = qvB \rightarrow$ only $v = E/B$ passes straight (**velocity selector**)

Force on a wire

- n electrons per volume, each $Bev_d \rightarrow$ $F = B(nAev_d)l\sin\theta = BIl\sin\theta$; $\vec{F} = I(\vec{l}\times\vec{B})$
- wire along B: **0**; $\perp B$: BIl, largest
- floating rod: $BIl = mg$ (50 g, 0.5 m, 0.2 T, g = 10 → 5 A)
- field reversed → force reversed

Torque on a coil



Rectangular coil PQRS in a uniform field; PQ and RS (length l) feel equal, opposite forces BIl; arm b sin θ.

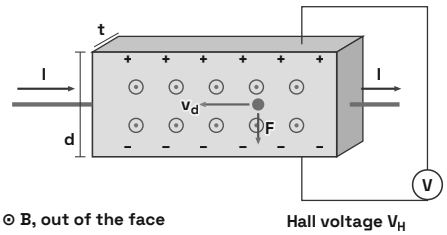
1. PQ, RS $\perp$ field: each $F = BIl$ per turn, opposite
2. QR, SP: equal, opposite, **same line** → no torque
3. couple: $\tau = BIl \times b\sin\theta = BIA\sin\theta$; N turns: $NBIA\sin\theta = NBIA\cos\alpha$
4. $\vec{\tau} = \vec{M}\times\vec{B}$, $M = NIA$

- plane along the field ($\alpha = 0$): largest, $NBIA$; plane $\perp$ field: **0** (moment along B, stable)
- net force on the coil **zero** in a uniform field; only a torque
- a coil = a magnet of moment NIA; energy $U = -MB\cos\theta$; 0° → 180° takes $W = 2MB$

Moving-coil galvanometer

- **principle**: torque on a current-carrying coil, balanced by a spring
- **radial field** (concave poles, soft-iron core): $\alpha = 0$ always → deflecting torque $NBIA$
- restoring torque $k\theta \rightarrow NBIA = k\theta \rightarrow I = G\theta$, $G = k/NBA$ → $I \propto \theta$: even scale
- without the radial field, torque $NBIA\cos\alpha$ falls; scale crowded
- current sensitivity NBA/k: more turns, stronger B, larger A, softer suspension (smaller k)
- voltage sensitivity NBA/kR: also small coil resistance R; more turns raise R too

Hall effect



Current along a slab in a field at right angles: electrons pushed to one face; Hall voltage V_H across width d; thickness t along B.

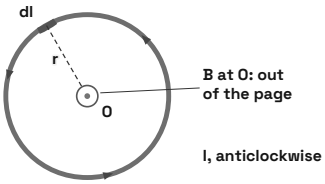
1. electrons feel $F_m = Bev_d$, collect on one face
2. field across: $E = V_H/d$, force eV_H/d
3. balance: $eV_H/d = Bev_d \rightarrow V_H = Bdv_d$
4. $v_d = I/(ne\,dt) \rightarrow V_H = \dfrac{BI}{net}$

- larger V_H: thin slab, **small** n (semiconductors); in copper n is huge → tiny V_H, hard to detect

- **Hall probe**: $B = V_H$ *net*$/I \to B \propto V_H$; face held $\perp$ field (largest reading); detects even the Earth's field

Biot–Savart law

- $dB \propto I, \propto dl, \propto \sin\theta, \propto 1/r^2 \to dB = \dfrac{\mu_0}{4\pi}\dfrac{I\,dl\sin\theta}{r^2}$
- direction $\perp$ element and r (thumb rule); on the element's own line: 0; inverse square like Coulomb's law



Circular coil, radius r, current I: every element at r, at 90° to the line to O; all dB along the axis.

Center of a coil: every dl at distance r, $\theta = 90°$, all dB one way $\to B = \dfrac{\mu_0 I}{4\pi r^2} \times 2\pi r = \dfrac{\mu_0 I}{2r}$; N turns $\mu_0 NI/2r$

- anticlockwise seen from a side $\to$ field out towards you (N face)

Axis of a coil

1. each element: $dB = \dfrac{\mu_0}{4\pi}\dfrac{I\,dl}{r^2}$, $r^2 = a^2 + x^2$
2. split: along the axis $dB\sin\varphi$ ($\sin\varphi = a/r$); across $dB\cos\varphi$
3. opposite elements: the parts **across cancel**
4. sum along: $\dfrac{\mu_0 I}{4\pi r^2} \cdot \dfrac{a}{r} \cdot 2\pi a = \dfrac{\mu_0 I a^2}{2r^3}$

- N turns: $B = \dfrac{\mu_0 N I a^2}{2(a^2+x^2)^{3/2}}$
- $x = 0 \to$ center value; $x \gg a \to \propto 1/x^3$ (a dipole, like a bar magnet); graph bell-shaped; off the axis $B \neq 0$
- 100 turns, 10 cm, 5 A, $x = 5$ cm: about 2.2 mT

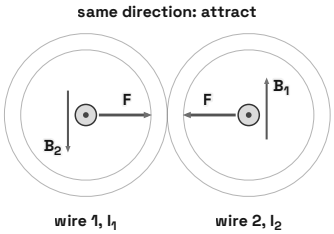
Straight wire: elements from $-\varphi_1$ to φ_2, $\int \cos\varphi\, d\varphi \to \dfrac{\mu_0 I}{4\pi a}(\sin\varphi_1 + \sin\varphi_2)$; infinite (90°, 90°) $\to \mu_0 I/2\pi a$; $B \propto 1/a$

Long solenoid: inside strong, **uniform**, along the axis; outside ≈ 0; $B = \mu_0 nI$ (not on the radius); ends ½

Ampère's circuital law

- $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enc}}$; any closed path; steady currents; choose a path with B constant and along it (or $\perp$)
- **wire**: circle of radius $r \to B \times 2\pi r = \mu_0 I$
- **solenoid**: rectangle abcd, ab $= l$ inside $\to$ only ab counts: $Bl = \mu_0 nlI \to B = \mu_0 nI$
- **toroid**: circle inside the windings threaded by N turns $\to B \times 2\pi r = \mu_0 NI$; outside and in the hole: 0; uniform only if thin
- with a core: $\times \mu_r$ (300 turns, 10 cm, μ_r 2000, 1 A $\to$ 1.2 T)

Parallel wires



Like currents: each wire in the other's field $B = \mu_0 I/2\pi r$; forces towards each other.

- $B_1 = \mu_0 I_1/2\pi r$ at wire 2 $\to F_{21} = B_1 I_2 l \to F/l = \mu_0 I_1 I_2/2\pi r$, towards wire 1; equal and opposite on wire 1 (Newton's third law)
- **like currents attract, unlike repel** (opposite of charges)
- $I_1 = I_2 = 1$ A, $r = 1$ m $\to 2 \times 10^{-7}$ N/m: the ampere
- midway between equal like currents: fields opposite $\to B = 0$
- AC wires: tiny force ($2 \times 10^{-7} I/d$ field), and it reverses many times a second $\to$ no visible movement

COMMON MISTAKES

✗ θ in $\phi = BA\cos\theta$ measured from the surface
✓ from the **normal**.

✗ 1 T $= 10^{-4}$ G
✓ 1 T = **10^4 G**; the oersted is a unit of H.

✗ a magnetic force speeds a charge up
✓ $\perp \vec{v}$: **no work**, speed unchanged.

✗ faster charge, longer period
✓ $T = 2\pi m/qB$: **same** for every speed.

✗ torque largest with the coil's plane $\perp B$
✓ largest with the plane **along** B.

✗ field off a coil's axis is zero
✓ not zero; only its formula is harder.

✗ like currents repel
✓ like currents **attract**.

✗ a toroid's field is uniform
✓ $\propto 1/r$; nearly uniform only when thin.

✗ large Hall voltage in metals
✓ small: n is large; semiconductors give large V_H.