Unit 14 · Electrical Circuits

STUDY NOTES

WHAT NEB'S GRADE 12 EXAM ASKS

- **CDC's grid (2078):** Electricity and Magnetism (Units 14–19 together): 21 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** MCQ: which junction fits $I_1 - I_2 = I_3 - I_4$ (currents in = currents out); long (8 marks, an OR): a student's potentiometer with the test cell reversed: why no balance and the fix, how to know the balance point, $E = 1.50 \times 25.0/37.5 = 1.00$ V, why it beats a voltmeter
- **MCQ (1 mark)**
  - I_4 at a junction
  - KVL's statement; KCL from charge
  - $E \propto l$ at balance
  - internal resistance from 650 and 520 cm
  - meter bridge balance point (25 cm)
  - potential gradient $k = I\rho/A$
  - $k = 2\text{ V} \div 4\text{ m}$
  - two cells in parallel: between the e.m.f.s
  - instrument for an e.m.f.: potentiometer
  - ideal voltmeter: infinite resistance
  - shunt for 10% through G
- **Short and long (5 or 8 marks, in parts)**
  - state KCL and KVL; their significance; KVL with a circuit diagram; AC and DC
  - Kirchhoff on a two-cell network: a current, a terminal p.d.
  - derive the Wheatstone bridge's balance condition
  - state the potentiometer's principle; potential gradient and its factors; why "voltmeter of infinite resistance"
  - derive r by potentiometer; E_1/E_2; a protective resistance; a 1 V driver
  - meter bridge: R, X doubled and swapped; G and cell interchanged; R and S from two null points
  - galvanometer → voltmeter; compare resistances of micro-, milli- and ammeters (voltmeters)
  - heater's power at 220 V; an ammeter across part of a wire (graph)

Key terms

Junction (node): point where three or more wires meet.

Branch: path between two junctions.

Kirchhoff's first law (junction rule, KCL): the algebraic sum of the currents meeting at a junction is zero.

Kirchhoff's second law (loop rule, mesh rule, KVL): in any closed loop, the algebraic sum of the e.m.f.s equals the algebraic sum of the potential drops (IR).

Wheatstone bridge: four resistances in a ring, a cell across one diagonal, a galvanometer across the other; finds an unknown resistance by a **null** (no deflection).

Meter bridge (slide wire bridge): a Wheatstone bridge whose two ratio arms are the two parts of a uniform 1 m wire.

Potentiometer: long uniform wire carrying a steady current; measures and compares e.m.f.s and internal resistance, drawing **no current** at balance.

Potential gradient k: fall of potential per unit length of the wire, V/l; V/m.

Galvanometer: detects small currents; moving-coil (motor effect), deflection ∝ current.

Shunt: low resistance in parallel with a galvanometer.

Superconductivity: total loss of resistance below a **critical temperature** T_c.

Perfect conductor: ideal, zero resistivity; no field and no p.d. inside.

Formulas

$\sum I = 0$	in +, out − **Use:** junction rule
$\sum E = \sum IR, \sum V = 0$	round a closed loop **Use:** loop rule
$\frac{P}{Q} = \frac{X}{R}$	$I_g = 0$ **Use:** balanced bridge
$X = \frac{l}{100 - l}R$	l in cm from A **Use:** meter bridge
$V = kl, k = \frac{I\rho}{A}$	steady I, uniform wire **Use:** potentiometer
$\frac{E_1}{E_2} = \frac{l_1}{l_2}$	same k **Use:** compare e.m.f.s
$r = \left(\frac{l_1}{l_2} - 1\right)R$	l_1 open, l_2 with R **Use:** internal resistance
$S = \frac{I_g}{I - I_g}G$	in parallel **Use:** ammeter of range I
$R = \frac{V}{I_g} - G$	in series **Use:** voltmeter of range V

Kirchhoff's laws

- first law ← **conservation of charge** (no charge piles up at a junction): current in = current out
- second law ← **conservation of energy** (electrostatic force conservative; potential has one value at a point)
- both hold for **DC and AC** (at every instant; with impedances)

Loop-rule signs (going round the loop)

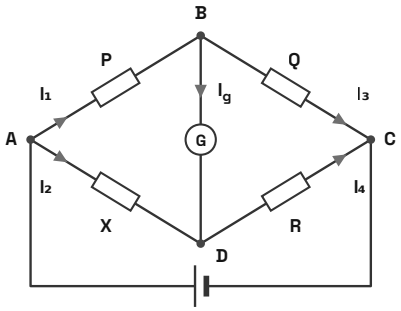
- cell − to +: **+E**; + to −: **−E**
- resistor with the current: **−IR**; against it: **+IR**

Solving a network

1. mark a current and direction in each branch; junction rule at once ($I_1 + I_2$ in the next branch)
2. choose as many loops as unknowns left, each with a new branch
3. go round each loop with the signs; sum = 0
4. solve together; **negative** answer → current flows the other way, same size

- each current's direction marked once; each loop's direction chosen afresh
- two cells in parallel, + to +: p.d. across the pair lies **between** the two e.m.f.s

Wheatstone bridge



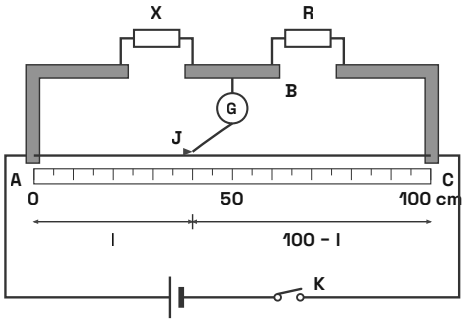
P in AB, Q in BC, R in CD, X in AD; cell across AC, galvanometer G across BD.

Derivation (Kirchhoff)

1. loop ABDA: $-I_1P - I_gG + I_2X = 0$
2. loop BCDB: $-I_3Q + I_4R + I_gG = 0$
3. junction B: $I_1 = I_g + I_3$; junction D: $I_g + I_2 = I_4$
4. balance, $I_g = 0$: $I_1P = I_2X$, $I_3Q = I_4R$, $I_1 = I_3$, $I_2 = I_4$
5. divide: $\frac{P}{Q} = \frac{X}{R}$

- B and D at the same potential; a **null method** (accurate: doesn't rely on a meter's scale)
- ratio arms P, Q of manganin or constantan (resistance hardly changes with temperature)
- balanced bridge: interchange the cell and G → **still no deflection**

Meter bridge



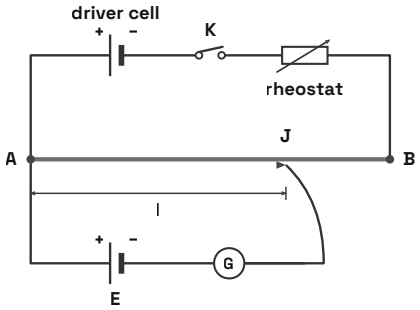
1 m constantan wire AC; X in the left gap, resistance box R in the right; G from the middle strip B to the jockey J; balance at l from A.

- wire uniform → resistance ∝ length → $P/Q = l/(100 - l)$ → $X = \frac{l}{100 - l}R$
- most accurate with balance near the **middle** → choose R near X
- wire: **constantan or manganin** (high resistivity, low temperature coefficient); thick copper strips at the ends
- 5 Ω left, 15 Ω right → $l = 25$ cm

Potentiometer

Principle: constant current through a uniform wire → p.d. across any length ∝ that length.

- proof: $V = IR = I\frac{\rho l}{A} = \left(\frac{I\rho}{A}\right) l = kl$
- k depends on the current (rheostat), resistivity, cross-section; 10 m of constantan or manganin, ten 1 m lengths



Driver cell, key and rheostat send a steady current through AB. Test cell's + to A (same end as the driver's +), − through G to the jockey; balance at J, length l.

- J too near A: p.d. AJ < E; too far: p.d. AJ > E; at balance no current in G **or the cell** → $E = kl$ (no Ir drop: the full e.m.f.)

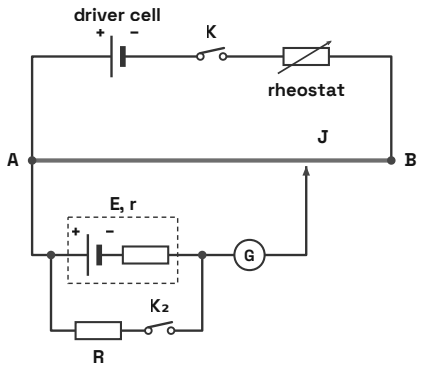
Two conditions for a balance point

1. driver's and test cell's + terminals at the **same end A**
2. p.d. across the whole wire > the e.m.f. measured (driver's e.m.f. larger)

- either fault: deflection **one way only**, wherever the jockey touches

Comparing e.m.f.s: both cells' + to A, − ends to a two-way key, common terminal through G to the jockey; $E_1 = kl_1$,

$E_2 = kl_2$, rheostat untouched $\rightarrow E_1/E_2 = l_1/l_2$; a **standard cell** gives the other



Internal resistance: the cell (E, r) as test cell, a resistance box R with key K_2 across its terminals.

Internal resistance

1. K_2 open: no current from the cell $\rightarrow E = kl_1$
2. K_2 closed: terminal p.d. $V = IR = kl_2$, $l_2 < l_1$
3. $E = I(R + r)$, $V = IR \rightarrow E/V = 1 + r/R$
4. $E/V = l_1/l_2 \rightarrow r = \left(\frac{l_1}{l_2} - 1\right) R$

- 650 and 520 cm, 8 Ω: $r = (650/520 - 1) \times 8 = 2\ \Omega$

Sensitivity: small p.d. change moves the balance far $\rightarrow$ **small** k: longer wire, or less current (more rheostat resistance)

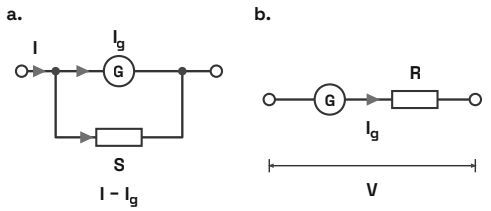
Potentiometer or voltmeter

- voltmeter takes current $\rightarrow$ reads terminal p.d. $E - Ir$
- potentiometer takes none at balance $\rightarrow$ the **e.m.f.**; an **ideal voltmeter** (infinite resistance)
- null method, balance read to 1 mm; also calibrates meters, measures tiny e.m.f.s (thermocouples)

Practical points

- uniform wire (else V not $\propto l$); steady driver cell, an accumulator (k fixed)
- current not left on (wire heats, ρ, k change); no copper (tiny k, high temperature coefficient)
- a high resistance (600 kΩ) beside the test cell: protects G and the cell far from balance; **doesn't move** the balance point (no current there); shorted out near balance for sensitivity

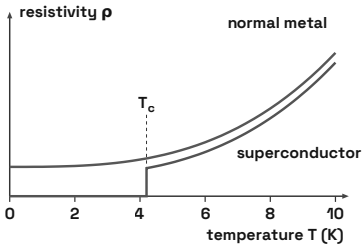
Galvanometer → ammeter, voltmeter



Ammeter (left): shunt S in parallel with G; I_g through G, $I - I_g$ through S. Voltmeter (right): high R in series; at V, current I_g.

- **ammeter**: in series, **low** resistance; G and S in parallel $\rightarrow$ $(I - I_g)S = I_gG \rightarrow S = \frac{I_g}{I - I_g}G$; whole meter $GS/(G + S) < S$
- **voltmeter**: in parallel, **high** resistance; $V = I_g(G + R) \rightarrow R = V/I_g - G$; whole meter $G + R$
- ideal ammeter **0**, ideal voltmeter ∞
- larger range $\rightarrow$ ammeter's resistance **smaller** (microammeter > milliammeter > ammeter); voltmeter's **larger** (voltmeter > millivoltmeter > microvoltmeter)
- uses of a shunt: protects G; makes an ammeter; across any resistance lowers it and raises the circuit's current
- 10% of the current through 99 Ω: $S = 99/9 = 11\ \Omega$

Superconductivity



Resistivity near 0 K: a normal metal levels off; a superconductor drops to zero at T_c.

- Kamerlingh Onnes, **1911**, mercury at **4.2 K** (liquid helium); a ring's current flowed over two years with no battery
- T_c: mercury 4.2 K, lead 7.2 K, niobium 9.2 K; Pu–Co–Ga alloy 18.5 K; none yet at room temperature
- **uses**: strong electromagnets (MRI: Nb–Ti at about 4 K), particle accelerators (LHC), fast electronics (SQUIDs, quantum computers), loss-free power cables (trials), maglev trains (603 km/h, 2015)
- superconductor is **more than** a perfect conductor: a perfect conductor keeps the field it had; a superconductor **pushes it out**
- **Meissner effect** (Meissner and Ochsenfeld, 1933): above T_c field lines pass through; below, they bend round it; surface currents cancel the field inside $\rightarrow B = 0$; **perfect diamagnet**, $\chi = -1$; a magnet floats over one in liquid nitrogen

Joule's laws of heating

Heat H in a conductor $\propto$

1. I^2 (R, t constant)
2. R (I, t constant)
3. t (I, R constant)

- $H = I^2Rt$ J $= I^2Rt/J$ cal; charge It falls through IR, loses $It \times IR$; $J = 4.18$ J/cal, only a conversion factor
- graphs: H–R, H–t, H–I^2 straight lines through O; H–I a parabola ($2 \times$ I $\rightarrow 4 \times$ H)
- constant R: $P = V^2/R \rightarrow$ 230 V to 220 V cuts power by 8.5%

COMMON MISTAKES

✗ currents into and out of a junction all taken +
✓ in +, out − (or the reverse), the same way throughout.

✗ a negative current is an error
✓ it flows **opposite** to the arrow, same size.

✗ the IR sign follows the real current
✓ it follows the current **marked** on the diagram.

✗ a voltmeter reads the e.m.f.
✓ it reads $E - Ir$; only a potentiometer at balance gives E.

✗ test cell's − to A
✓ its + to A, the same end as the driver's +.

✗ a superconductor is just a perfect conductor
✓ it also **expels** the field (Meissner).

Science Playground, Katunje, Bhaktapur
https://science-playground.com/learn/grade-12-physics/electrical-circuits