Unit 12 · Diffraction

STUDY NOTES

WHAT NEB'S GRADE 12 EXAM ASKS

- **CDC's grid (2078):** Waves and Optics (Units 6–13 together): 15 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** an MCQ: red light replaced by white in a diffraction pattern: the central maximum white, the side bands coloured, violet inside and red outside
- **MCQ (1 mark)**
  - first minimum: $\sin\theta = \lambda/d$
  - slit halved → central maximum twice as wide
  - red in place of blue → wider pattern
  - resolving power of a microscope, of a telescope; what raises each
  - why light seems to go straight
- **Short and long (5 or 8 marks, in parts)**
  - define diffraction; when it is noticeable; vs interference
  - Fresnel vs Fraunhofer, with diagrams
  - single slit: derive the minima; the pattern, with a diagram; why the centre is wider and brighter
  - width of the central maximum
  - grating: element, equation, number of orders, uses
  - resolving power, Rayleigh's criterion
  - numericals: $\sin\theta = n\lambda/d$, $2\lambda D/d$, $d\sin\theta = n\lambda$, $1.22\lambda/D$

Key terms

Diffraction: bending and spreading of a wave at an edge or through a gap, into the geometrical shadow.

Fresnel diffraction: source or screen at a finite distance; curved wavefronts at the slit; no lenses.

Fraunhofer diffraction: source and screen effectively at infinity; plane wavefronts; two lenses.

Central maximum: the bright band straight ahead, from the first minimum on one side to the other.

Angular width: the angle the central maximum spans at the slit, $2\lambda/d$.

Diffraction grating: many equally spaced parallel slits (300–1200 per mm); rulings opaque.

Grating element: $a + b$, slit plus ruling, the repeat distance; $= 1/N$.

Resolving power: ability to show two close objects as separate; reciprocal of the limit of resolution.

Airy disc: the bright centre of a point's image through a round aperture.

Rayleigh's criterion: just resolved when the central maximum of one image falls on the first minimum of the other.

Numerical aperture: $\mu\sin\theta$ of a microscope objective.

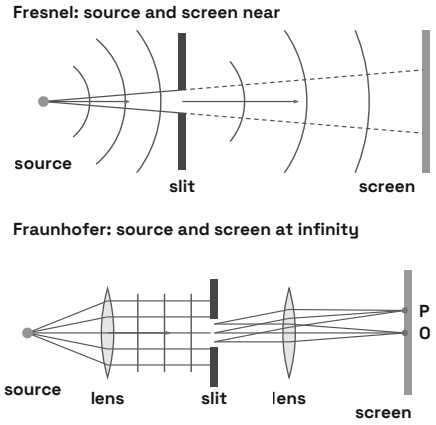
Formulas

$d\sin\theta_n = n\lambda$	$n = 1, 2, \ldots$ **Use:** single slit minima
$d\sin\theta_n = (2n+1)\frac{\lambda}{2}$	approximate **Use:** secondary maxima
$x_1 = \frac{\lambda D}{d}$	$D \approx f$ with a lens **Use:** first minimum
$2x_1 = \frac{2\lambda D}{d}$	angle $\frac{2\lambda}{d}$ **Use:** central maximum
$(a+b)\sin\theta_n = n\lambda$	$a + b = 1/N$ **Use:** grating maxima
$n_{max} < \frac{a+b}{\lambda}$	whole number below **Use:** orders seen
$\theta = \frac{1.22\lambda}{D}$	round aperture **Use:** Airy disc
$d = \frac{\lambda}{2\mu\sin\theta}$	NA $= \mu\sin\theta$ **Use:** microscope limit
$\frac{2\mu\sin\theta}{\lambda}$	oil, short λ **Use:** microscope RP
$d\theta = \frac{1.22\lambda}{D}$	angle, radians **Use:** telescope limit
$\frac{D}{1.22\lambda}$	larger objective **Use:** telescope RP

Diffraction

- every wave: water round a harbour wall, sound round a door, light round a razor's edge
- gap $\gg \lambda$: hardly spreads, sharp shadow; gap comparable to λ: spreads clearly
- light $\lambda < 1\ \mu$m: rarely noticed; sound $\lambda \approx 1$ m: hear round a wall, not see
- X-rays (10^{-10} m) and ribs: far too wide apart; crystals' atoms do diffract them
- evidence that light is a wave; light does bend, only a little

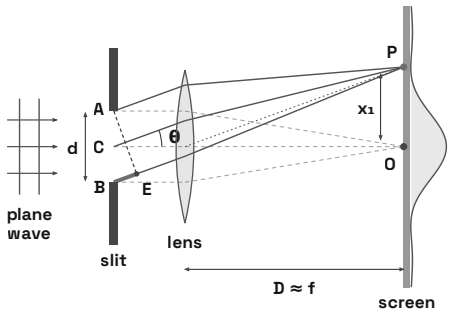
	Interference	Diffraction
waves from	two coherent sources	one wavefront's different parts
fringes	equal width	central one twice as wide
brightness	all bright fringes equal	falls fast away from the centre
dark fringes	quite dark	not perfectly dark



Top: Fresnel, curved wavefronts, source and screen near. Bottom: Fraunhofer, plane wavefronts, lenses.

	Fresnel	Fraunhofer
source, screen	finite distance	at infinity
wavefront at slit	spherical, cylindrical	plane
lenses	none	two (or a laser, far screen)
pattern	depends on distances	fixed shape, only size changes

Single slit

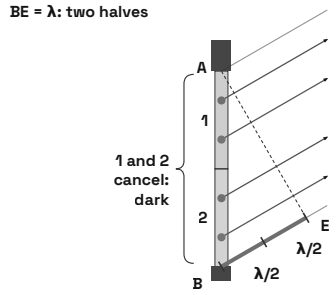


Slit AB of width d; AE $\perp$ the rays, so BE $= d \sin\theta$; straight ahead to O, at θ to P.

- O: every wavelet the same path (a lens adds none) $\rightarrow$ central maximum
- P: edges differ by $BE = d \sin\theta$

Minima

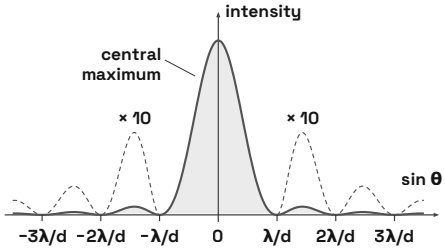
1. $BE = \lambda$: halves AC, CB; matching points $d/2$ apart differ by $\lambda/2 \rightarrow$ cancel in pairs $\rightarrow$ dark
2. $BE = 2\lambda$: four parts, cancel in pairs $\rightarrow$ dark
3. $BE = n\lambda$: $2n$ parts $\rightarrow d \sin\theta_n = n\lambda$, $n = 1, 2, 3\ldots$



First minimum: two halves, each pair $\lambda/2$ apart in path, cancel.

Secondary maxima

1. $BE = 3\lambda/2$: three parts; two cancel, one left over $\rightarrow$ faint bright
2. $BE = 5\lambda/2$: five parts, one left $\rightarrow$ fainter
3. $d \sin\theta_n = (2n+1)\lambda/2$; really 1.43λ, $2.46\lambda\ldots$



Intensity against $\sin\theta$: zeros at $\pm\lambda/d$, $\pm 2\lambda/d$; side peaks 4.7 %, 1.6 % (dashed ×10).

- centre: light from the whole slit $\rightarrow$ brightest, **twice as wide**; side bands from one leftover part $\rightarrow$ fainter, equal widths

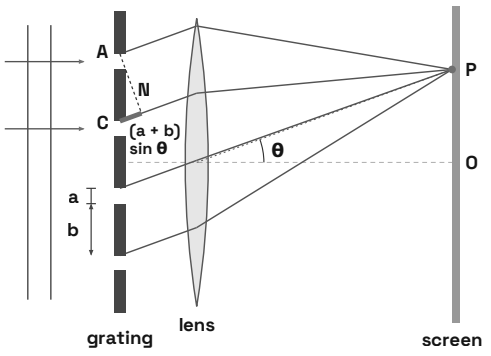
- same-looking formula: single slit $d \sin\theta = n\lambda$ **dark**; Young's **bright**

Width of the central maximum

1. $\tan\theta = x_1/D$; first minimum $\sin\theta = \lambda/d$
2. small θ: $x_1/D = \lambda/d \to x_1 = \lambda D/d$ ($= \lambda f/d$ with a lens)
3. width $= 2x_1 = 2\lambda D/d$; angular width $2\lambda/d$

- $\propto 1/d$: halve the slit, double the band; $\propto \lambda$: red wider; $\propto D$, angle the same
- white light: white centre, coloured sides, violet inside, red outside
- under water: $\lambda \to \lambda/\mu$, about 75 % $\to$ pattern shrinks to 75 %
- 500 nm, 0.1 mm, 2 m: $2\lambda/d = 0.01$ rad, width 2.0 cm

Diffraction grating



Neighbouring slits' wavelets at θ differ by $(a+b)\sin\theta$; a whole number of $\lambda \to$ bright at P.

1. corresponding points A, C of neighbouring slits, $a + b$ apart
2. AN $\perp$ the rays: $CN = (a+b)\sin\theta$, the same for every neighbouring pair
3. $CN = n\lambda$: all slits in step $\to$ principal maximum, $\sin\theta_n = Nn\lambda$

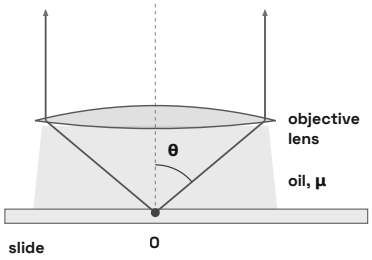
- 5000 lines/cm: $a + b = 2$ μm; 500 nm $\to d/\lambda = 4$: 4th order at 90°, so **3** orders seen
- 6000 lines/cm, 500 nm, 2nd order: $\sin\theta = 0.6$, $\theta = 36.9°$
- between orders nearly all light cancels ($N_s - 1$ dark directions): sharp lines; the two-slit rule $(2n+1)\lambda/2$ gives only some of the dark ones
- white light: white zeroth order, a spectrum each side; **red turned most** (prism: violet most)
- precise: large angles, $\lambda = d\sin\theta/n$ with no μ needed; spectrometers, stars' make-up
- transmission and reflection gratings; CD tracks 1.6 μm, DVD 0.74 μm

Resolving power

- a point's image is an Airy disc and rings: no lens makes a point
- just resolved: brightness dips about 26 % between the peaks



Two images: well apart, resolved; at Rayleigh's limit, a small dip; closer, one peak.



The objective's cone from O, semi-angle θ; oil of index μ raises $\mu\sin\theta$.

- microscope: wider cone, **immersion oil** ($\mu \approx 1.5$), blue or UV; electron microscope, far shorter λ
- some books: $0.61\lambda/\mu\sin\theta$ (Rayleigh) for Abbe's $\lambda/2\mu\sin\theta$; both $\propto \lambda/\text{NA}$
- telescope: a **larger objective** (also gathers more light); magnification enlarges, doesn't sharpen
- 0.2 m, 550 nm: $d\theta = 3.4 \times 10^{-6}$ rad

COMMON MISTAKES

✗ light can't bend
✓ it does; its small λ makes the bending small.

✗ single-slit maxima exactly at $(2n+1)\lambda/2$
✓ roughly; really 1.43λ, 2.46λ.

✗ $d\sin\theta = n\lambda$ gives bright bands for one slit
✓ **dark**; bright in Young's.

✗ a narrower slit gives a narrower centre
✓ **wider**: $\propto 1/d$.

✗ grating: $d/\lambda = 4$, so 4 orders
✓ the 4th is at 90°: **3** seen.

✗ a grating bends violet most
✓ **red**; a prism, violet.

✗ more magnification resolves more
✓ only a larger aperture or shorter λ.

✗ $\lambda = 6000$ Å $= 6 \times 10^{-9}$ m
✓ 6×10^{-7} m.