Unit 11 · Interference of Light

STUDY NOTES

WHAT NEB'S GRADE 12 EXAM ASKS

- **CDC's grid (2078):** Waves and Optics (Units 6–13 together): 15 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** nothing from this unit; its optics MCQ is on diffraction (Unit 12), which uses this unit's path differences
- **MCQ (1 mark)**
  - coherent sources: same f, constant phase difference
  - phase at the nth bright fringe ($2n\pi$); $\varphi = \frac{2\pi}{\lambda}\Delta x$
  - path difference at the first dark fringe ($\lambda/2$)
  - $\lambda/2 \to \pi$; $2\lambda \to 0$ (720°)
  - fringe width when λ, d or D change
- **Short and long (5 or 8 marks, in parts)**
  - coherent sources; why two bulbs aren't
  - superposition; conditions for sustained interference
  - constructive and destructive, by path difference
  - optical path; temporal and spatial coherence
  - YDSE: complete the diagram; derive the bright and dark positions and $\beta = \lambda D/d$
  - what changes with d, D, water, white light; why the single slit
  - numericals: β from λ, D, d

Key terms

Interference: two or more overlapping waves adding by superposition: brighter in some places, darker in others.

Coherent sources: same frequency, **constant phase difference**; made from one source (equal amplitudes not required).

Constructive interference: in phase, path difference $n\lambda$, amplitude $a_1 + a_2$.

Destructive interference: in antiphase, path difference $(2n+1)\lambda/2$, amplitude $|a_1 - a_2|$.

Sustained interference: a pattern that stays still and lasts, so it can be seen and measured.

Coherence length: length of a wave train, $\approx \lambda^2/\Delta\lambda$; white light ~1 μm, laser cm or more.

Fringe visibility: $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$, 1 for black dark fringes.

Optical path: the distance light would go in a vacuum in the same time, μd.

Path difference: difference of the distances (optical paths) two waves have travelled.

Fringe width β: distance between the centres of neighboring bright (or dark) fringes.

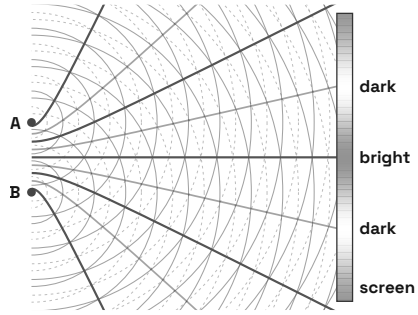
Formulas

$\varphi = \frac{2\pi}{\lambda}\Delta x$	$\lambda \leftrightarrow 2\pi$ **Use:** phase from path
$\Delta x = n\lambda,\ \varphi = 2n\pi$	bright **Use:** constructive
$\Delta x = (2n+1)\frac{\lambda}{2}$	$\varphi = (2n+1)\pi$ **Use:** destructive
$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\varphi$	resultant amplitude **Use:** two waves
$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi$	equal: $4I_0$ and 0 **Use:** intensity
$s = \mu d$	slab adds $(\mu - 1)d$ **Use:** optical path
$\Delta x = d\sin\theta \approx \frac{xd}{D}$	small θ **Use:** YDSE path
$x_n = \frac{n\lambda D}{d}$	$n = 0$ bright centre **Use:** bright fringes
$x_n = \frac{(2n+1)\lambda D}{2d}$	halfway between **Use:** dark fringes
$\beta = \frac{\lambda D}{d}$	each band $\beta/2$ **Use:** fringe width
$\beta' = \beta/\mu$	$\lambda \to \lambda/\mu$ **Use:** in a liquid
$\sin\theta = \frac{n\lambda}{d}$	any angle **Use:** angle of a maximum

Interference around us

- soap bubbles, oil on a wet road, peacock feathers, lens coatings

- thin film: rays from the top and bottom surfaces interfere; reflection off the outer surface flips the wave (π) → a very thin film is **black**
- rainbow: **dispersion**, not interference (only the faint supernumerary bows are)



Crests solid, troughs dashed: crest on crest along the bright lines, crest on trough along the dark.

Coherence

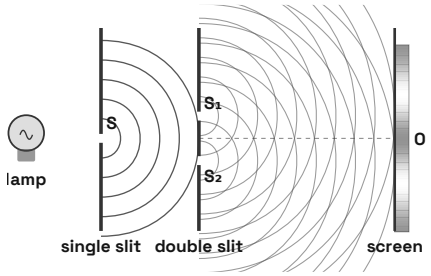
- two bulbs: atoms emit bursts of random phase lasting nanoseconds, many frequencies → pattern changes millions of times a second → even glow
- laser: one frequency, steady phase → the best coherent source
- **dividing the wavefront**: Young's slits, Lloyd's mirror, Fresnel's biprism and double mirror, Billet's split lens
- **dividing the amplitude**: thin films, Michelson and Rayleigh interferometers
- path difference $\lambda/2 \to \pi \to$ dark; $5\lambda/2 \to$ dark; λ, 2λ... → bright
- equal waves: bright $4I_0$, dark 0; energy moved, not lost

Sustained interference needs

1. coherent, monochromatic sources (from one source)
2. path difference within the coherence length (temporal coherence)
3. a narrow source (spatial coherence): Young's single slit
4. nearly equal amplitudes, for contrast
5. the same polarization
6. slits narrow and close, screen far
7. a steady setup

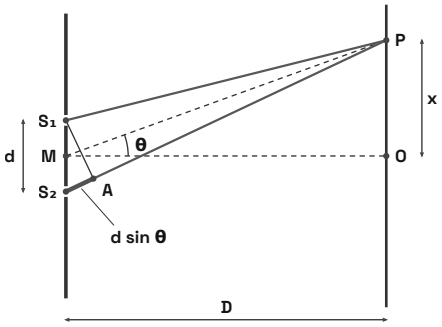
- optical path: in time d/v, vacuum light goes $c\,d/v = \mu d$; counts the waves; glass 6 μm (μ 1.5) adds 3 μm, about 6 fringes

Young's double slit



Lamp, single slit S, double slit S_1 S_2 (coherent, in phase), screen with fringes; centre O bright.

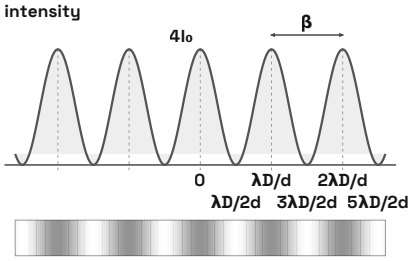
- the single slit gives one wavefront to both slits; S_1, S_2 equidistant from S → in phase



$S_1A \perp S_2P$: path difference $S_2A = d\sin\theta$; $\tan\theta = x/D$.

Fringe positions

1. $\Delta x = S_2P - S_1P = S_2A = d\sin\theta$
2. θ small: $\sin\theta \approx \tan\theta = x/D \to \Delta x = xd/D$
3. bright: $xd/D = n\lambda \to x_n = n\lambda D/d$; $n = 0$: centre bright
4. dark: $xd/D = (2n+1)\lambda/2 \to x_n = (2n+1)\lambda D/2d$
5. $\beta = x_{n+1} - x_n = \lambda D/d$, the same for bright and dark, every order



Intensity $\propto \cos^2(\pi xd/\lambda D)$: maxima $4I_0$, zeros halfway; every fringe β wide.

- $\beta \propto \lambda$ (red wider than blue), $\propto D$, $\propto 1/d$
- 2.0 mm fringes; D 1.0 → 1.5 m and d halved: $\times 1.5 \times 2 =$ 6.0 mm
- 0.5 mm, 2 m, 600 nm: $\beta =$ 2.4 mm; sodium → blue: narrower
- in water (1.33): 4.0 mm → 3.0 mm
- slits wider apart → fringes closer, can't be seen; slits as close as $\lambda \to$ few, faint fringes
- constant extra phase at one slit → the whole pattern shifts, unchanged
- slightly different frequencies → fringes drift, no steady pattern; one source brighter → dark fringes not dark
- **white light**: white centre; a few coloured fringes, violet inside, red outside; then an even whitish glow

COMMON MISTAKES

✗ coherent = same amplitude and in phase
✓ same frequency, **constant** phase difference.

✗ dark fringes at path difference $(2n+1)\lambda$
✓ $(2n+1)\lambda/2$.

✗ a bright band is β wide
✓ β is a bright **and** a dark band; each $\beta/2$.

✗ the centre is dark
✓ always bright (path difference 0).

✗ larger d, wider fringes
✓ narrower: $\beta \propto 1/d$.

✗ rainbows are interference
✓ dispersion; only supernumerary bows are.

✗ 450 nm $= 4.5 \times 10^{-8}$ m
✓ 4.5×10^{-7} m.