Unit 8 · Waves in Pipes and Strings

STUDY NOTES

WHAT NEB'S GRADE 12 EXAM ASKS

- **CDC's grid (2078):** Waves and Optics (Units 6–13 together): 15 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** a short question (5 marks): what harmonics are; a flute 30 cm long, open at both ends, $v = 330$ m/s: its first three harmonics (550, 1100, 1650 Hz); the same flute played on a moving bus (Unit 9, Doppler)
- **MCQ (1 mark)**
  - closed pipe's fundamental $v/4l$
  - true of an open pipe
  - cause of the end correction
  - closed pipe: 3rd harmonic = 1st overtone
  - string: length doubled, tension × 4
  - centre of a string in its fundamental (antinode)
  - resonances 300 and 500 Hz → fundamental 100 Hz
  - pipe length from λ
- **Short (5 marks, in parts)**
  - harmonics vs overtones in closed and open pipes
  - stationary waves in pipes; how the ends set nodes and antinodes
  - derive fundamental and overtones of closed and open pipes
  - why a closed pipe has only odd harmonics; quality
  - end correction; effective length
  - laws of a stretched string; sketch the third harmonic
  - numericals: pipe lengths, harmonics, string frequency and speed

Key terms

Organ pipe: a tube whose air column vibrates when blown; **open** (both ends open) or **closed** (one end closed).

Fundamental frequency f_1: the lowest frequency a string or air column vibrates at.

Harmonics: whole-number multiples of the fundamental, nf_1; the first harmonic is the fundamental.

Overtones: frequencies above the fundamental that a system actually produces; the fundamental isn't one.

Mode: one way of vibrating, with its own number of nodes and antinodes.

End correction e: distance of the antinode beyond an open end; $e = 0.3d = 0.6r$ (Rayleigh).

Effective length: the length plus the end correction(s).

String: a perfectly flexible, thin wire of the same diameter all along.

Linear mass density μ: mass per unit length, m/l (kg/m).

Sonometer: a wire on two bridges over a hollow box, tensioned by a hanging weight; tests the string laws.

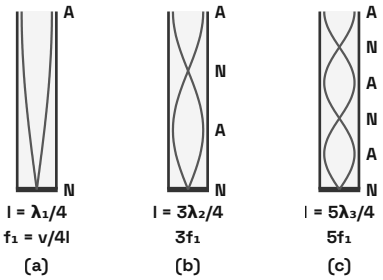
Formulas

$f_1 = \frac{v}{4l}$	$\lambda_1 = 4l$ **Use:** closed pipe
$f_n = (2n-1)f_1$	$1:3:5$ **Use:** closed pipe's modes
$f_1 = \frac{v}{2l}$	$\lambda_1 = 2l$ **Use:** open pipe
$f_n = nf_1$	$1:2:3$ **Use:** open pipe's modes
$f = \frac{v}{4(l+e)}$	$e = 0.3d$ **Use:** closed, end correction
$f = \frac{v}{2(l+2e)}$	two open ends **Use:** open, end correction
$v = \sqrt{T/\mu}$	$\mu = m/l$ **Use:** wave on a string
$\mu = \pi r^2 \rho$	ρ density of the wire **Use:** wire's μ
$f_1 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$	both ends nodes **Use:** string
$f_n = nf_1$	$1:2:3$ **Use:** string's modes
$f \propto \frac{1}{l}, \sqrt{T}, \frac{1}{\sqrt{\mu}}$	the other two fixed **Use:** string laws

Harmonics and overtones

- fundamental 100 Hz, all present: harmonics 100, 200, 300…; overtones 200 (1st), 300 (2nd)…
- "1st overtone = 2nd harmonic" only when every harmonic is present (string, open pipe)
- closed pipe: 1st overtone = **3rd** harmonic, 2nd overtone = 5th
- first find which harmonics the system makes, then count the overtones among them

Closed pipe

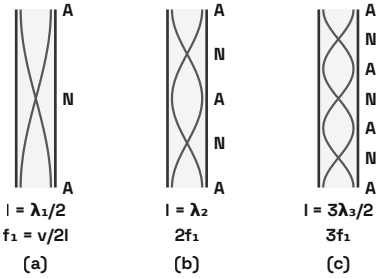


First three modes, closed at the bottom: node N at the closed end, antinode A at the open end.

1. fundamental: $\frac{\lambda_1}{4} = l \rightarrow f_1 = \frac{v}{4l} = \frac{1}{4l}\sqrt{\frac{\gamma P}{\rho}}$
2. second mode: $\frac{3\lambda_2}{4} = l \rightarrow f_2 = \frac{3v}{4l} = 3f_1$ (3rd harmonic, 1st overtone)
3. third mode: $\frac{5\lambda_3}{4} = l \rightarrow f_3 = 5f_1$ (5th harmonic, 2nd overtone)

- nth mode $(2n-1)f_1$; mth overtone $(2m+1)f_1$
- **even harmonics missing** → hollow, poorer quality
- closed end can't move (node); between the ends 1, 3, 5… quarter wavelengths
- 150 Hz closed pipe can't make 300 Hz
- test tube, resonance tube, stopped organ pipes (a narrow-necked bottle is a Helmholtz resonator, not a pipe)

Open pipe



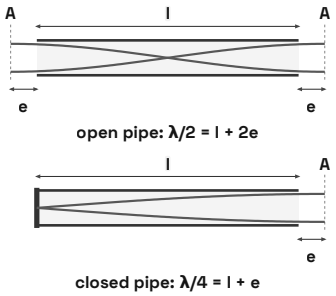
First three modes, open at both ends: antinodes A at both ends.

1. fundamental: $\frac{\lambda_1}{2} = l \rightarrow f_1 = \frac{v}{2l}$
2. second mode: $\lambda_2 = l \rightarrow f_2 = 2f_1$ (2nd harmonic, 1st overtone)
3. third mode: $\frac{3\lambda_3}{2} = l \rightarrow f_3 = 3f_1$ (3rd harmonic, 2nd overtone)

- nth mode nf_1; mth overtone $(m+1)f_1$
- **all harmonics** → richer quality (flute, bansuri)

- same length: open pipe's fundamental **twice** the closed pipe's (an octave higher)
- longer pipe → longer λ → lower pitch
- flute 30 cm, open, $v = 330$: $f_1 = 330/0.6 = 550$ Hz, then 1100, 1650 Hz
- open pipe's 2nd harmonic at 680 Hz: $l = \lambda_2 = 340/680 = 0.5$ m

End correction

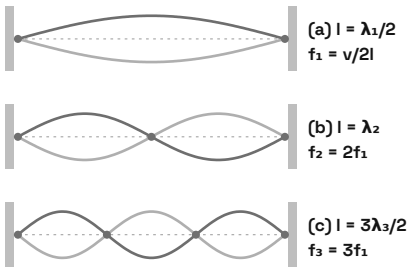


Antinode e beyond each open end: open $\lambda/2 = l + 2e$, closed $\lambda/4 = l + e$.

- the air's vibration reaches a little beyond the open end → acoustic length > physical length
- open pipe: add e at both ends; closed: once
- same length, wider pipe → lower note (r is added to l: falls, not $\propto 1/r$)
- effective length $= l + e$: 0.75 m + 0.03 m = 0.78 m

Stretched strings

- tension the elastic factor, μ the inertial factor (as $\sqrt{E/\rho}$)
- same steel and tension, twice the diameter: $\mu \times 4$ → waves at half the speed
- plucked: waves reflect at both fixed ends → **both ends nodes**



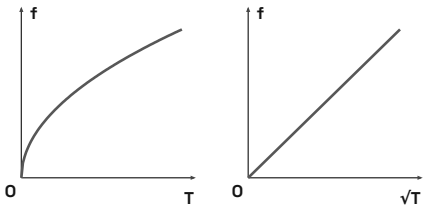
First three modes: 1, 2, 3 loops, each $\lambda/2$; third harmonic: 4 nodes, 3 antinodes.

1. one loop: $\lambda_1 = 2l \to f_1 = \frac{1}{2l}\sqrt{T/\mu}$
2. two loops: $\lambda_2 = l \to f_2 = 2f_1$
3. three loops: $\lambda_3 = \frac{2l}{3} \to f_3 = 3f_1$

- all harmonics → rich sound; a plucked string sounds several at once
- 2nd overtone at 300 Hz → $f_1 = 100$ Hz

Laws of a stretched string

1. **length**: $f \propto 1/l$ (μ, T fixed): a fret shortens the string, the note rises; half the length, an octave up
2. **mass**: $f \propto 1/\sqrt{\mu}$ (l, T fixed): thick bass strings play low
3. **tension**: $f \propto \sqrt{T}$ (l, μ fixed): tightening a peg raises the note; $4\times$ tension, $2\times f$



f against T rises ever more slowly; against $\sqrt{T}$, a straight line through the origin.

- graphs: f vs $1/l$, vs $1/\sqrt{\mu}$, vs $\sqrt{T}$: straight lines through the origin
- sonometer: bridges moved until the wire resonates with a fork (the paper rider jumps off)

COMMON MISTAKES

✗ closed pipe's 1st overtone is the 2nd harmonic
✓ the **3rd**: only odd harmonics.

✗ closed pipe's third mode $3\lambda_3/4 = l$
✓ $5\lambda_3/4 = l, f_3 = 5f_1$.

✗ the antinode is exactly at the open end
✓ $e = 0.3d$ beyond it.

✗ $f \propto 1/r$ for a wider pipe
✓ the note falls, but r is only added to l.

✗ a string's ends are antinodes
✓ fixed ends are **nodes**.

✗ $f \propto T$
✓ $f \propto \sqrt{T}$.

✗ law of tension keeps m constant
✓ keeps μ (mass per length) and l constant.