Unit 7 · Mechanical Waves

STUDY NOTES

WHAT NEB'S GRADE 12 EXAM ASKS

- **CDC's grid (2078):** Waves and Optics (Units 6–13 together): 15 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** nothing from this unit by itself; its flute question (Unit 8) uses $v = f\lambda$ with the speed of sound
- **MCQ (1 mark)**
  - what sets a wave's speed; fastest medium
  - the formula for a gas; Laplace's correction
  - temperature, pressure, humidity
  - f doubled: λ halves, v the same
  - 343 m/s at 20 °C → 0 °C
  - sonar and echoes (go and back)
  - hydrogen to oxygen ($v \propto 1/\sqrt{M}$)
- **Short (5 marks, in parts)**
  - solids, liquids, gases: why solids are fastest
  - derive Newton's formula; why Laplace corrected it
  - effect of temperature, pressure, humidity, molar mass
  - numericals: $\sqrt{K/\rho}$, $\sqrt{Y/\rho}$, $\sqrt{\gamma RT/M}$, v at another temperature, wavelength

Key terms

Mechanical wave: a disturbance that needs a material medium, passed on by its particles.

Speed of a wave: distance travelled in unit time; $v = f\lambda$.

Elastic property: the medium's resistance to deformation, which pulls particles back (modulus E).

Inertial property: the medium's mass, which must be set moving (density ρ).

Bulk modulus K: pressure change ÷ volumetric strain, $-dP/(dV/V)$.

Young's modulus Y: stress ÷ strain along a rod.

Compression / rarefaction: air squeezed to a little higher / stretched to a little lower pressure.

Isothermal: at constant temperature (Newton's assumption).

Adiabatic: no heat exchanged (Laplace's, the right one).

Adiabatic index γ: C_P/C_V; 1.67 monatomic, 1.40 diatomic.

Humidity: water vapor in the air.

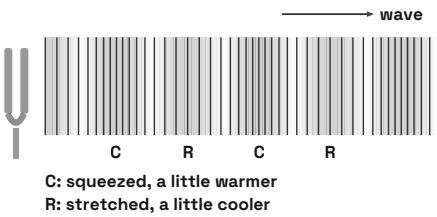
Formulas

$v = f\lambda$	f from the source, v from the medium **Use:** any wave
$v = \sqrt{E/\rho}$	elastic ÷ inertial **Use:** mechanical wave
$v = \sqrt{T/\mu}$	μ mass per length **Use:** string
$v = \sqrt{Y/\rho}$	Young's modulus of the rod **Use:** solid rod
$v = \sqrt{K/\rho}$	bulk modulus **Use:** liquid or gas
$v = \sqrt{P/\rho}$	isothermal, $K = P$; 280 m/s **Use:** Newton
$v = \sqrt{\gamma P/\rho}$	adiabatic, $K = \gamma P$; 331.6 m/s **Use:** Laplace
$v = \sqrt{\gamma RT/M}$	T in K, M in kg/mol **Use:** in terms of T
$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$	kelvin **Use:** two temperatures
$v \approx 331 + 0.6t$	t in °C, near room temperature **Use:** rough rule

What sets the speed

- f from the source, v from the medium, $\lambda = v/f$: into water, same pitch, λ over 4 times longer

- stiffer → faster; denser → slower
- air 343, water ≈ 1480, steel rod ≈ 5100, granite ≈ 6000 m/s
- steel 6500 × denser than air but over a million times stiffer → about 15 × faster
- tension × 4 → string's wave speed × 2



Compressions (C) warm a little, rarefactions (R) cool; the wave moves on before heat can flow.

Newton and Laplace

Newton's formula

1. assumed isothermal: $PV = \text{constant}$
2. $P\,dV + V\,dP = 0 \to P = -\frac{dP}{dV/V} = K$
3. $v = \sqrt{K/\rho} = \sqrt{P/\rho}$
4. air at STP: $\sqrt{1.013 \times 10^5 / 1.29} = 280$ m/s; measured ≈ 331 m/s: 15% too low

Laplace's correction

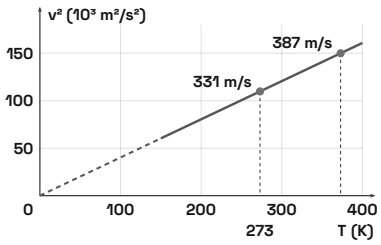
1. sound is fast and air conducts heat badly → no time for heat to flow: adiabatic
2. $PV^\gamma = \text{constant} \to \gamma P V^{\gamma-1} dV + V^\gamma dP = 0$
3. $\div V^{\gamma-1}$: $\gamma P = -\frac{dP}{dV/V} = K$
4. $v = \sqrt{\gamma P/\rho}$; air: $\sqrt{1.4 \times 1.013 \times 10^5 / 1.29} = 331.6$ m/s ✓

- it is about the swings **inside** the wave ($K = \gamma P$), not the air's overall temperature

Temperature, pressure, humidity

Temperature

1. one mole: $PV = RT$, $V = M/\rho \to P/\rho = RT/M$
2. $v = \sqrt{\gamma RT/M} \to v \propto \sqrt{T}$ (γ, R, M fixed)

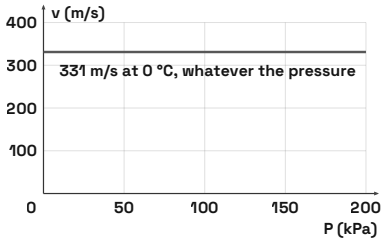


v^2 against T in kelvin: a straight line through the origin.

- warmer air, faster molecules ($v \approx 68\%$ of c_{rms})
- $dv/dT = v/2T \approx 0.61$ m/s per K: the $331 + 0.6t$ rule; for big changes use $\sqrt{T}$
- same T: $v \propto 1/\sqrt{M}$: hydrogen 4 × oxygen ($\sqrt{32/2}$); helium raises a voice's pitch

Pressure

1. constant T: $P/\rho = RT/M = \text{constant}$
2. $v = \sqrt{\gamma \times \text{constant}}$: unchanged



Speed against pressure at 0 °C: a horizontal line.

- double the pressure → double the density
- Everest: thin air no slower for being thin; slower for being cold, −30 °C → 312 m/s

Humidity

- water vapor (18 g/mol) lighter than air (29) → moist air less dense → sound a little faster
- tiny: saturated air at 20 °C only 0.4% faster
- sound heard far on a still night: cool air near the ground under warmer air bends it down (temperature, not humidity)

COMMON MISTAKES

✗ higher pressure, faster sound
✓ no change at constant T: ρ rises in step.

✗ Laplace corrected for the air's temperature
✓ for adiabatic swings inside the wave, $K = \gamma P$.

✗ $v^2 \propto t$ in °C
✓ in **kelvin**.

✗ frequency doubled → speed doubled
✓ λ halves; v fixed by the medium.

✗ echo distance = vt
✓ $vt/2$: there and back.

✗ sound carries at night because of humidity
✓ mainly the cool air near the ground.

✗ a rod's formula uses a string's properties
✓ Young's modulus and density of the rod's material.