Unit 6 · Wave Motion

STUDY NOTES

WHAT NEB'S GRADE 12 EXAM ASKS

- **CDC's grid (2078):** Waves and Optics (Units 6–13 together): 15 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** nothing from this unit by itself; its pipe and string question (Unit 8) needs nodes, antinodes and $\lambda/2$ from here
- **MCQ (1 mark)**
  - what is true of a progressive wave
  - equation of a wave towards $+x$
  - condition for a stationary wave
  - wave speed: on the medium; f doubled at fixed λ
  - node to node $\lambda/2$; string in two loops
  - v, λ, f from an equation or $v = f\lambda$
  - stationary amplitude $2a$
- **Short (5 marks, in parts)**
  - define a progressive wave; derive its equation in its forms
  - the meaning of each term of $y = a\sin(\omega t - kx)$
  - state superposition; derive the stationary wave equation
  - nodes and antinodes: where, and why they don't travel
  - progressive vs stationary
  - numericals: read a, λ, f, v from an equation; add two waves; strings with loops

Key terms

Wave: a disturbance that travels through a medium, carrying energy and momentum, while each particle only vibrates about its own place.

Mechanical wave: needs a material medium (sound, ripples, seismic waves); **electromagnetic**: needs none; **matter wave**: goes with moving particles.

Transverse wave: particles vibrate at right angles to the travel (rope, ripples).

Longitudinal wave: particles vibrate along the travel, making **compressions** and **rarefactions** (sound).

Progressive wave: travels on in one direction at a definite speed; same amplitude everywhere, phase changing gradually.

Wave number k: $2\pi/\lambda$, radians of phase per meter (rad/m).

Phase difference: how far one particle's vibration is ahead of or behind another's.

Principle of superposition: where waves overlap, the resultant displacement is the vector sum of the separate ones.

Stationary wave: two equal waves (amplitude, speed, frequency) in opposite directions; the pattern doesn't travel, no energy passes.

Node: point of a stationary wave always at rest; **antinode**: point of greatest amplitude, $2a$.

Loop: the part between two neighboring nodes.

Formulas

$y = a\sin(\omega t - kx)$	towards $+x$; $+kx$ towards $-x$ **Use:** progressive wave
$y = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$	same wave **Use:** form with T
$y = a\sin\frac{2\pi}{\lambda}(vt - x)$	same wave **Use:** form with v
$k = \frac{2\pi}{\lambda}$, $\omega = 2\pi f$	rad/m, rad/s **Use:** wave number
$v = \frac{\omega}{k} = f\lambda$	read ω, k off the equation **Use:** wave speed
$\phi = \frac{2\pi}{\lambda}x$	x path difference **Use:** phase difference

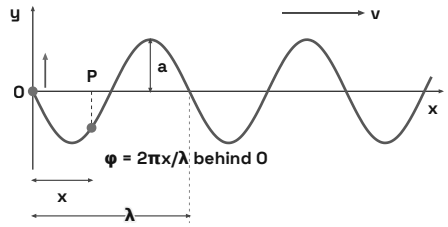
$y = y_1 + y_2 + \ldots$	add with signs **Use:** superposition
$y = 2a\cos kx\,\sin\omega t$	amplitude $A = 2a\cos kx$ **Use:** stationary wave
$x = \frac{n\lambda}{2}$	$n = 0, 1, 2 \ldots$ **Use:** antinodes
$x = (2n+1)\frac{\lambda}{4}$	$n = 0, 1, 2 \ldots$ **Use:** nodes
$\frac{\lambda}{2}$	node–antinode $\frac{\lambda}{4}$ **Use:** node to node
$y = -2a\sin kx\,\cos\omega t$	node at $x = 0$ **Use:** fixed end

Waves

- leaf on a pond bobs up and down; it isn't carried (energy travels, matter doesn't)
- each particle repeats the motion of the one before it, a little later: the key to the equation
- earthquake: P waves longitudinal (first), S waves transverse

Progressive wave

- same frequency and amplitude for every particle; phase changes gradually; no particle permanently at rest



P, a distance x from O, lags O by $\phi = 2\pi x/\lambda$.

The equation

1. O: $y = a\sin\omega t$
2. P gets the disturbance x/v later: $y = a\sin(\omega t - \phi)$
3. λ of path $\leftrightarrow 2\pi$ of phase $\rightarrow \phi = \frac{2\pi}{\lambda}x$
4. $y = a\sin(\omega t - kx)$, $k = 2\pi/\lambda$
5. $\omega = 2\pi/T \rightarrow a\sin 2\pi(\frac{t}{T} - \frac{x}{\lambda})$; $\omega = 2\pi v/\lambda \rightarrow a\sin\frac{2\pi}{\lambda}(vt - x)$

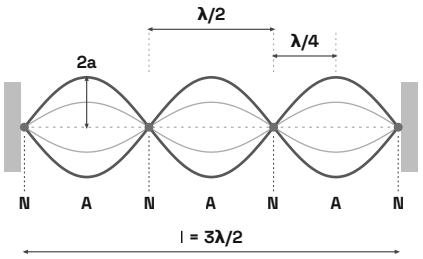
- towards $-x$: P gets it **before** O, leads: $a\sin(\omega t + kx)$
- direction: x and t with **opposite** signs $\rightarrow +x$; same sign $\rightarrow -x$
- $a\sin(kx - \omega t)$ also goes towards $+x$: half a cycle from $a\sin(\omega t - kx)$
- reading an equation: $y = 0.03\sin(40t + 5x)$: $a = 0.03$ m, $k = 5$, $\lambda = 2\pi/5$ m, $v = 40/5 = 8$ m/s towards $-x$
- v set by the medium; f doubled at the same λ would need double v

Characteristics

1. carries energy in the direction of travel
2. goes on in one direction, unchanged in an ideal medium
3. same amplitude for every particle
4. no nodes or antinodes
5. particles oscillate about their own places
6. phase changes gradually
7. transverse or longitudinal

Superposition and the stationary wave

- waves pass through each other unchanged; only where they overlap do displacements add (crest 3 cm + trough 2 cm $\rightarrow$ 1 cm)
- same direction, same f: interference; opposite directions: stationary wave; slightly different f: beats
- conditions: same amplitude, speed and frequency, **opposite** directions (made by reflection)
- examples: guitar, sitar, sarangi strings (transverse); flutes, organ pipes, resonance tube (longitudinal)



String fixed at both ends, at five instants: nodes N never move; antinodes A swing to $\pm 2a$.

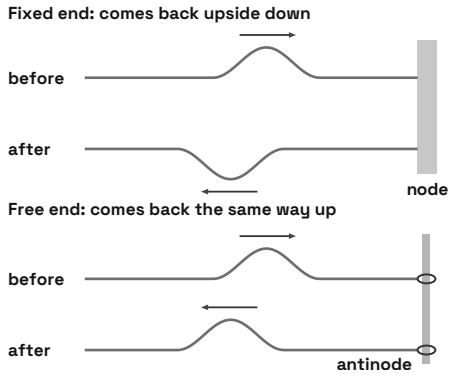
The equation

1. $y_1 = a\sin(\omega t - kx)$, $y_2 = a\sin(\omega t + kx)$
2. $y = y_1 + y_2$; $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$
3. $y = 2a\cos kx\,\sin\omega t$: every particle in SHM at the same f, amplitude $2a\cos kx$ set by the place

- no $(\omega t - kx) \rightarrow$ nothing moves along: the pattern stands
- antinodes: $\cos kx = \pm 1 \rightarrow x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda \ldots$
- nodes: $\cos kx = 0 \rightarrow x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4} \ldots$
- one loop in phase; neighboring loops in **opposite** phase (π)

- all particles pass the mean together: the string is straight twice a period
- energy stays in each loop, swapping KE and PE
- sound: pressure changes **most at displacement nodes**, least at antinodes
- two nodes measured apart → $\lambda/2$ (resonance tube)
- antinode amplitude: each wave 3 mm, λ 40 cm, 10 cm ($\lambda/4$) from an antinode → a node, 0

The ends



Fixed end: pulse returns upside down, a node. Free end: returns the same way up, an antinode.

- **free (open) end**: displacement reflected without phase change → $2a\cos kx$, antinode at $x = 0$
- **fixed (closed) end**: phase change π, $y_2 = -a\sin(\omega t + kx) \to -2a\sin kx\cos\omega t$, node at $x = 0$
- in a pipe "node" means displacement: closed end a displacement node = pressure antinode; open end the reverse

	Progressive	Stationary
energy	carried along	stays in each loop
amplitude	same everywhere	0 at nodes to $2a$
phase	changes gradually	same in a loop; π in the next
nodes	none	$\lambda/2$ apart
pattern	moves at v	stays
straight	never all at once	twice a period
equation	$a\sin(\omega t \pm kx)$	$2a\cos kx\sin\omega t$

COMMON MISTAKES

✗ the medium travels with the wave
✓ particles only vibrate about their places.

✗ $a\sin(kx - \omega t)$ goes towards $-x$
✓ towards $+x$: x and t have opposite signs.

✗ nodes are λ apart
✓ $\lambda/2$; node to antinode $\lambda/4$.

✗ stationary amplitude is a
✓ up to $2a$ at the antinodes.

✗ a stationary wave carries energy along
✓ none passes a node.

✗ all particles of a stationary wave are in phase
✓ in phase in a loop, opposite in the next.

✗ higher frequency, faster wave
✓ v set by the medium; λ shrinks.