# Unit 3 · Fluid Mechanics

STUDY NOTES

**WHAT NEB'S GRADE 12 EXAM ASKS**

- **CDC's grid (2078):** Mechanics (Units 1–3 together): 13 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** MCQs: air blown under a balance pan (Bernoulli: the pan goes down); capillary rise on the Moon ($6h$); short: define surface tension, relate it to surface energy, the terminal velocity of two raindrops that join; or state and derive Bernoulli's equation and explain the thumb over a hose
- **MCQ (1 mark)**
  - highest surface tension; what isn't surface tension
  - capillary: $h$ on the Moon; $h$ with half the radius
  - upthrust in free fall
  - Torricelli: outflow rate; height from $v$
  - density ratio from fractions under water
  - pipe's diameter doubled: $v \div 4$
  - Poiseuille: what changes $V$ most ($r$)
  - what gives a wing its lift
- **Short and long (5 or 8 marks, in parts)**
  - derive: pressure at depth $h$; $W = T\Delta A$; capillary rise; terminal velocity; continuity; Bernoulli
  - state: Archimedes, Newton's law of viscosity, Stokes, Poiseuille, Bernoulli
  - explain: heavy ship bottom, Archimedes in a satellite, round drops, hot soup, contact angle
  - numericals: iron in a liquid (tension), iceberg fraction, ice block and a man, plate on water, drop into droplets, viscous force on a plate, terminal velocity, flow rate, pressure in a narrowing pipe

## Key terms

**Fluid**: flows; takes its container's shape; layers slide under the smallest sideways force.

**Pressure**: normal force per unit area, $P = F/A$; pascal (Pa = $N/m^2$).

**Gauge pressure**: pressure above atmospheric, $h\rho g$.

**Pascal's law**: pressure applied to an enclosed fluid is transmitted **undiminished** to every part of the fluid and the walls.

**Specific gravity**: density ÷ density of water at 4 °C; no unit, no dimensions.

**Upthrust**: upward force of a fluid on a body in it = apparent loss of weight.

**Archimedes' principle**: a body wholly or partly immersed in a fluid has an upthrust equal to the weight of fluid displaced.

**Law of flotation**: a floating body displaces its own weight of liquid.

**Metacenter M**: where the vertical through the new center of buoyancy meets the body's center line; GM = metacentric height.

**Surface tension** $T$: force per unit length of a line drawn in the surface, at right angles to it.

**Surface energy**: extra PE of surface molecules; per unit area = work to increase the area by one unit.

**Cohesion / adhesion**: attraction between like / unlike molecules.

**Angle of contact**: between the solid and the tangent to the liquid surface at the contact point, measured **inside** the liquid.

**Capillarity**: rise or fall of a liquid in a narrow tube or pores, from surface tension and adhesion.

**Viscosity**: a fluid's property of opposing relative motion between its layers.

**Coefficient of viscosity** $\eta$: tangential force per unit area for unit velocity gradient.

**Streamline (steady) flow**: every particle passing a point has the same velocity there; **streamline**: tangent = velocity; two never cross.

**Turbulent flow**: above the **critical velocity**; eddies, velocity at a point keeps changing.

**Terminal velocity**: steady speed when drag + upthrust = weight.

## Formulas

| | |
|---|---|
| $P = P_0 + h\rho g$ | any vessel's shape<br>**Use:** pressure at depth $h$ |
| $F_2 = F_1 A_2/A_1$ | moves $A_2/A_1$ times less<br>**Use:** hydraulic lift |
| $U = V\sigma g$ | $\sigma$ fluid's density<br>**Use:** upthrust |
| $v/V = \rho/\sigma$ | fraction under<br>**Use:** floating |
| $T = F/L;\ W = T\,\Delta A$ | N/m = $J/m^2$, $[MT^{-2}]$<br>**Use:** surface tension, energy |
| $2TL\cos\theta = mg$ | needle of length $L$<br>**Use:** needle on water |
| $h = \frac{2T\cos\theta}{r\rho g}$ | $hr$ constant (Jurin)<br>**Use:** capillary rise |

| | |
|---|---|
| $F = -\eta A \frac{dv}{dx}$ | Pa s; $[ML^{-1}T^{-1}]$<br>**Use:** Newton's law of viscosity |
| $R_e = \frac{\rho v D}{\eta}$ | $< 2000$ laminar, $> 3000$ turbulent<br>**Use:** Reynolds number |
| $V = \frac{\pi P r^4}{8\eta l}$ | $V$ volume per second<br>**Use:** Poiseuille |
| $F = 6\pi\eta r v$ | small sphere, slow<br>**Use:** Stokes' law |
| $v = \frac{2r^2(\rho-\sigma)g}{9\eta}$ | $\rho$ sphere, $\sigma$ liquid<br>**Use:** terminal velocity |
| $A_1 v_1 = A_2 v_2$ | $Av$ m$^3$/s<br>**Use:** continuity |
| $P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$ | along a streamline<br>**Use:** Bernoulli |
| $v = \sqrt{2gh}$ | open tank, depth $h$<br>**Use:** Torricelli |

## Pressure, Pascal, Archimedes

- column: $m = Ah\rho$, weight $Ah\rho g$ on area $A \to P = h\rho g$: area cancels; $P \propto h, \propto \rho$
- same depth, same pressure in any vessel; equal in all directions; always normal to a surface
- 1 cm of mercury = 13.6 cm of water; air: Bhaktapur ≈ 86 kPa, Muktinath ≈ 63 kPa
- hydrometer sinks deeper in a less dense liquid; **lactometer**: watered milk → sinks deeper
- **upthrust**: top $(P_0 + h_1\sigma g)A$ down, bottom $(P_0 + h_2\sigma g)A$ up $\to U = (h_2 - h_1)\sigma g A = V\sigma g$
- $U$ depends only on the volume under and the fluid's density, not the body or its depth; iron sinks in water, floats in mercury
- sink: $W > U$; float: $W < U$ when fully under; $W = U$: stays put (fish, submarine)
- ice in sea water: 920/1030 = 89 % under
- in free fall or an orbiting satellite: no apparent weight → no upthrust

### Floating body's stability

1. weight down through G; upthrust up through B (center of the displaced liquid)
2. tilted: B moves to B′ on the deeper side; the two forces make a couple
3. **M above G: stable**; M below G: capsizes; M at G: neutral (a round log)

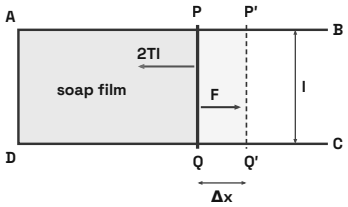
- ship's bottom made heavy, passengers sit: keeps G low, below M
- in a stable ship B is usually **below** G

## Surface tension and surface energy

- surface acts as a stretched membrane, shrinks to the **smallest area**; gases have none
- $T$ falls as temperature rises and with impurities (soap); salt raises it a little
- low $T$: antiseptics spread into wounds; soap wets cloth; camphor dances; hot soup tastes better
- examples: water strider, floating needle, paintbrush hairs cling, thread loop on a film → circle

**Molecular theory** (range ≈ $10^{-9}$ m, sphere of influence)

1. inside: pulled equally → no net force
2. near or on the surface: net pull **inwards**
3. → surface molecules have more PE → area kept least → small drops are **spheres** (least area for a volume)




Film on a frame, sliding wire PQ of length $l$: two surfaces pull with $2Tl$.

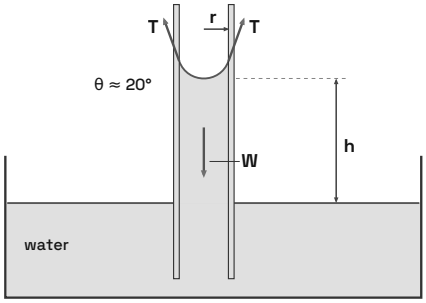
### $W = T\,\Delta A$: derivation

1. two surfaces: $F = 2Tl$
2. pull out by $\Delta x$: $W = 2Tl\,\Delta x$
3. new area $\Delta A = 2l\,\Delta x \to W = T\,\Delta A$, $T = W/\Delta A$

- surface energy per unit area = $T$ (at constant temperature)
- spraying a drop into droplets: more area → work needed; two drops joining: $R = 2^{1/3}r$, energy falls (ratio $2^{1/3}$)

## Angle of contact and capillarity

- adhesion > cohesion: wets, concave, $\theta < 90°$ (water–glass ≈ 0°, kerosene 26°)
- cohesion > adhesion: doesn't wet, convex, $\theta > 90°$ (mercury–glass ≈ 140°, water on wax > 100°)
- $\theta$ depends on: the solid and liquid; cleanliness; the medium above; temperature; impurities




Capillary of radius $r$: the glass pulls the rim with $T$ per length at $\theta$; the upward part holds the column $h$.

### Capillary rise: derivation

1. contact circle $2\pi r$; upward pull $2\pi r T\cos\theta$
2. column's weight $\pi r^2 h\rho g$ (meniscus neglected)
3. equal → $h = \frac{2T\cos\theta}{r\rho g}$

- $h \propto 1/r$; $h \propto 1/g$ (Moon: $6h$); $\theta > 90° \to$ depression (mercury)
- examples: pen nib, towel, lamp wick, soil, blotting paper, sugar cube, damp walls

- **not** capillarity: blood in capillaries (the heart pumps), a thermometer (expansion); in tall trees transpiration does the main lift

## Viscosity and flow

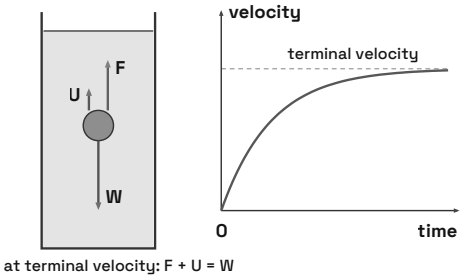
- layers: still at a bed or wall (**no-slip**), faster away from it; **velocity gradient** $dv/dx$
- $F \propto A$, $F \propto dv/dx \to F = -\eta A\, dv/dx$; minus: against relative motion
- 1 poise = 1 dyne s/cm$^2$ = 0.1 Pa s; water $1.0 \times 10^{-3}$ Pa s
- warming: liquid's $\eta$ **falls** (honey runs, cold engine drags); gas's $\eta$ **rises**
- shear stress = $\eta \times$ velocity gradient (a solid: modulus $\times$ strain)
- laminar: below critical velocity, layers don't mix, drag $\propto v$; turbulent: eddies, drag $\propto v^2$
- critical velocity $v_c = 2000\eta/\rho D$

### Poiseuille

- assumptions: steady laminar flow along the axis; same pressure across a section; liquid at rest at the wall
- by dimensions: $V = k\eta^a r^b (P/l)^c \to a = -1, b = 4, c = 1$; $k = \pi/8$
- $r$ doubled $\to$ 16 $\times$ flow; arteriole narrowed 16 % $\to$ flow halved
- uses: blood flow, drips and needles, lubrication, viscometers
- limits: laminar only; straight narrow tube; Newtonian fluid; no slip; steady (blood pulses)

## Stokes' law and terminal velocity

- $F = 6\pi\eta r v$ (by dimensions: $F = k\eta r v$); small, smooth sphere, slow, laminar



at terminal velocity: F + U = W

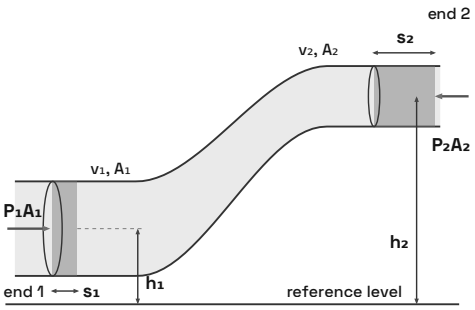
Weight down; upthrust and drag up; speed levels off at the terminal velocity.

1. $W = \frac{4}{3}\pi r^3 \rho g$; $U = \frac{4}{3}\pi r^3 \sigma g$; $F = 6\pi\eta r v$
2. $F + U = W \to 6\pi\eta r v = \frac{4}{3}\pi r^3(\rho - \sigma)g$
3. $v = \frac{2r^2(\rho-\sigma)g}{9\eta}$

- $v \propto r^2$ (straight $v$–$r^2$ line); $\rho < \sigma$: rises (bubble); two raindrops join: $R = 2^{1/3} r \to v \times 2^{2/3} \approx 1.59$
- uses: $\eta$ by falling sphere $\eta = \frac{2r^2(\rho-\sigma)g}{9v}$; settling tanks; ESR; paints; dust chambers; soil texture; Millikan's oil drop
- clouds float, parachutes land safely, hail falls slowly: drag grows with speed

## Continuity and Bernoulli

- continuity (mass conserved): in $\Delta t$, $\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$ $\to Av$ constant; narrow $\to$ fast
- uses: hose nozzle, thumb over a hose; blood fast in a narrowed artery, slow in capillaries; ducts; jet nozzles; turbines; gorges
- energy per unit volume: KE $\frac{1}{2}\rho v^2$, PE $\rho g h$, **pressure energy** $P$ (push $\Delta m$ in: $PA\,\Delta l \div A\,\Delta l\,\rho = P/\rho$ per kg)



Tube of flow: end 1 ($P_1$, $v_1$, $A_1$, $h_1$), end 2 ($P_2$, $v_2$, $A_2$, $h_2$).

**Bernoulli: derivation** (incompressible, non-viscous, steady)

1. $A_1 s_1 = A_2 s_2 = \Delta V$
2. work on the fluid: $P_1 \Delta V - P_2 \Delta V$
3. = gain in KE $\frac{1}{2}\rho \Delta V (v_2^2 - v_1^2)$ + PE $\rho \Delta V g(h_2 - h_1)$
4. $\to P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

- horizontal: $P + \frac{1}{2}\rho v^2$ constant: **faster $\to$ lower pressure**
- Torricelli: surface and hole both at $P_0$, surface $v \approx 0 \to v = \sqrt{2gh}$; a sealed tin stops flowing
- venturimeter: $P_1 - P_2 = \rho g h \to V = A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$

### Applications

1. wing's lift: faster air above $\to$ lower pressure; the wing also turns air down
2. helicopter rotor
3. sprayer, atomizer, carburetor, nebulizer
4. Magnus effect: spinning ball curves to its fast side
5. venturimeter; Venturi mask
6. roofs lifted in storms; pulled towards a fast train; chimney in wind; air under a balance pan $\to$ pan goes down

### COMMON MISTAKES

✗ surface tension in N/m$^2$, viscosity in N s/m
✓ $T$ in **N/m**; $\eta$ in **N s/m$^2$**.

✗ floating is stable when B is above G
✓ when the **metacenter** M is above G.

✗ capillaries carry blood by capillarity
✓ the heart pumps it; thermometers work by expansion.

✗ viscosity of every fluid falls when heated
✓ liquids' falls; gases' **rises**.

✗ lift because top air must "meet" bottom air
✓ no equal transit; faster air above, air turned down.

✗ $h$ independent of $g$
✓ $h \propto 1/g$: $6h$ on the Moon.

✗ Torricelli in a sealed tank
✓ needs the top open to the air.