Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Over the wall

ANSWER HA · 6

Tests: the path's equation with a point put in, a quadratic in $\tan\theta$ (using $\sec^2\theta = 1 + \tan^2\theta$); two angles, or none; the range formula used where the landing point is not level with the start (the tempting answer).

a. Path: $y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$. With $x = 30$, $y = 10$: $\frac{9.8 \times 900}{2 \times 625} = 7.056$, so $10 = 30\tan\theta - 7.056(1 + \tan^2\theta)$, that is $7.056\tan^2\theta - 30\tan\theta + 17.056 = 0$ ✓.

b. $\tan\theta = \frac{30 \pm \sqrt{900 - 4 \times 7.056 \times 17.056}}{2 \times 7.056} = 0.676$ or 3.576: θ **= 34.1° or 74.4°**. Time to the window $t = \frac{30}{25\cos\theta}$: **1.45 s** at 34.1° (just past the top), **4.46 s** at 74.4° (coming down).

c. The equation for general u: $\frac{4410}{u^2}\tan^2\theta - 30\tan\theta + \frac{4410}{u^2} + 10 = 0$. Real angles need the discriminant ≥ 0: $900 \geq 4 \cdot \frac{4410}{u^2}\left(\frac{4410}{u^2} + 10\right)$, which gives $u^2 \geq 407.9$: **at least about 20.2 m/s**. $\frac{u^2}{g}$ is the greatest range back down to the starting level; the window is 10 m higher, so the ball needs more speed to get there.

MARKING SCHEME

- ✓ The path's equation with $\sec^2\theta = 1 + \tan^2\theta$.
- ✓ The given quadratic.
- ✓ 34.1° and 74.4°.
- ✓ 1.45 s and 4.46 s.
- ✓ The discriminant condition: $u \geq 20.2$ m/s.
- ✓ The range formula holds only for landing at the starting level.

2 Off a cliff

ANSWER HA · 5

Tests: a projectile landing below its start, where the time comes from a quadratic; that 45° is best only when the landing is level with the start (the tempting answer).

a. Taking up as positive, the stone lands when $-20 = 15t\sin\theta - 4.9t^2$.

45°: $4.9t^2 - 10.61t - 20 = 0$, $t = 3.374$ s; across $15\cos 45° \times 3.374 =$ **35.8 m**.

30°: $4.9t^2 - 7.5t - 20 = 0$, $t = 2.926$ s; across $15\cos 30° \times 2.926 =$ **38.0 m**.

b. Falling the extra 20 m gives the stone extra time in the air whatever the angle, and in that time a flatter throw moves across faster ($u\cos\theta$ larger). So it pays to give up some height for horizontal speed: the best angle from a height is below 45° (here about 31°, giving 38.0 m).

MARKING SCHEME

- ✓ The landing equation $-20 = 15t\sin\theta - 4.9t^2$.
- ✓ 45°: $t = 3.37$ s, 35.8 m.
- ✓ 30°: $t = 2.93$ s, 38.0 m.
- ✓ The drop adds time at every angle.
- ✓ So a larger horizontal speed (a lower angle) wins.

3 The scale in the lift

ANSWER

Tests: the scale reads the normal reaction, not the weight; Newton's second law with the reaction and the weight; acceleration upward can mean slowing down while going down (the tempting answer: "going up").

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a. Reaction $R = 70 \times 9.8 = 686$ N, weight $60 \times 9.8 = 588$ N. $R - mg = ma$: $a = \frac{98}{60} = \mathbf{1.63\ m/s^2}$ **upward**. Not certain: the lift may be **speeding up going up or slowing down going down**; both are an upward acceleration. A · 5

b. Steady speed: $a = 0$, $R = mg$: **60 kg**.

c. Falling freely, $a = g$ downward: $mg - R = mg$, $R = 0$: **0 kg**; he and the scale fall together, and it doesn't push on him at all.

MARKING SCHEME

- ✓ $R - mg = ma$.
- ✓ $a = 1.63$ m/s^2 upward.
- ✓ Either rising faster or falling slower.
- ✓ 60 kg at steady speed.
- ✓ 0 in free fall.

4 A bullet in wood

ANSWER A · 5

Tests: a steady resistance from a stopping distance ($v^2 = u^2 - 2as$), with impulse as a check; that speed doesn't fall in proportion to distance (the tempting answer).

a. $0 = 400^2 - 2a(0.1)$: $a = 800\,000$ m/s^2. $F = ma = 0.02 \times 800\,000 = \mathbf{16\,000\ N}$. $t = \frac{400}{800\,000} = \mathbf{0.0005\ s}$.

b. Impulse $Ft = 16\,000 \times 0.0005 = 8$ N s $= mu - 0 = 0.02 \times 400$ ✓.

c. $v^2 = 400^2 - 2 \times 800\,000 \times 0.05 = 160\,000 - 80\,000 = 80\,000$: $v = \mathbf{283\ m/s}$. The wood takes away the same **energy** ($F \times$ distance) for each centimeter, so half the wood takes half the kinetic energy, v^2 halves, and the speed falls only by the factor $\frac{1}{\sqrt{2}}$.

MARKING SCHEME

- ✓ $a = 800\,000$ m/s^2, $F = 16\,000$ N.
- ✓ $t = 0.0005$ s.
- ✓ The impulse 8 N s equals the momentum lost.
- ✓ $v = 283$ m/s.
- ✓ Half the wood takes half of v^2 (the energy), not half the speed.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	0	10	11	**21**

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