Unit 16 · Linear Programming Problems

WORKSHEET

Full marks 24

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 24

1 Key words [3]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

corner	**determinant**	**optimal**	**pivot column**	**pivot row**	**slack**	**surplus**	**tableau**

To turn $2x + y \leq 16$ into an equation, add a _____ variable, the amount of the resource left unused. The equations and the objective row are set out in a _____. The entering variable's column, the _____, has the most negative entry of the bottom row; the _____ has the smallest ratio of the right side to a positive entry. Each tableau shows one _____ of the feasible region, and when the bottom row has no negative entry the solution is _____.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. A slack variable can never be negative. ☐

ii. In the first tableau for $Z = 3x + 5y$, the bottom row holds 3 and 5. ☐

iii. The pivot row is the row with the largest ratio. ☐

iv. A negative entry in the pivot column gives no ratio. ☐

v. If the pivot column has no positive entry at all, Z has no maximum. ☐

vi. At the start of the method, x and y are the basic variables. ☐

3 Multiple choice [5]

Choose the correct option for each of the following questions.

i. With the constraint $3x + 2y \leq 12$ written as $3x + 2y + s_1 = 12$, what does $s_1 = 4$ mean? [1]

a. 4 units of the resource were used
b. 4 units of the resource were left unused
c. the constraint is broken by 4
d. Z is 4

ii. The bottom row of a tableau is $(-2, -5, 0, 0 \mid 0)$ under x, y, s_1, s_2. Which variable enters? [1]

a. x
b. y
c. s_1
d. none: the tableau is optimal

iii. The pivot column has entries 3, −1 and 2, with right sides 12, 4 and 6. Which row is the pivot row? [1]

a. the first (ratio 4)
b. the second (ratio −4)
c. the third (ratio 3)
d. the second (ratio 4)

iv. A final tableau's bottom row is $(0, 0, 3, 1 \mid 23)$ under x, y, s_1, s_2, with $y = 2$ and $x = 3$ in the last column. The maximum of Z is: [1]

a. 3, at $x = 3$

b. 4, the sum of the slack entries

c. 23, at $(3, 2)$

d. 23, at $(2, 3)$

v. Maximizing $Z = x + 2y$ with $x - y \leq 2$ and $-2x + y \leq 2$, after one pivot the bottom row is $(-5, 0, 0, 2 \mid 4)$ and the column of x has entries -1 and -2. What should you conclude? [1]

a. the maximum is 4

b. the maximum is at $x = 0$

c. Z has no maximum (the problem is unbounded)

d. pivot on the -2

4 Short answers [10]

Write short answers to the following questions.

a. Explain why the pivot row is chosen by the **smallest** ratio, using the constraints $2x + y \leq 16$ and $x + y \leq 10$ as x enters with $y = 0$. [2]

b. Wangmo's first tableau for "maximize $Z = 4x + 3y$ subject to $x + 2y \leq 10$, $3x + y \leq 15$" is below. Find the two mistakes, and write the correct tableau. [3]

Basic	x	y	s_1	s_2	RHS
s_1	1	2	1	0	10
s_2	3	1	1	1	15
Z	4	3	0	0	0

c. Using the simplex method, maximize $Z = 5x + 4y$ subject to $x + y \leq 5$, $2x + y \leq 8$, $x, y \geq 0$. **[5]**

Answer: _____

5 Think **[3]**

In the final tableau of the last question, the bottom row has 3 under s_1 and 1 under s_2. Rojina says, "So one more unit on the right of $x + y \leq 5$ would raise the maximum by 3." Check his claim by solving $x + y = 6$, $2x + y = 8$, and say what the 3 and the 1 mean.

6 Activity: Corners by hand, corners by tableau (not marked)

You need: graph paper and a ruler. Draw the feasible region of the numerical question: $x + y \leq 5$, $2x + y \leq 8$, $x, y \geq 0$.

Corner	(x, y)	$Z = 5x + 4y$	Visited by the simplex method?

Mark the corners the tableaux showed, in order, with arrows. Did the method visit every corner?