Unit 16 · Linear Programming Problems

HARDER PRACTICE QUESTIONS

Full marks 20

Write your answers on separate paper. Show your working, with every tableau and the row operations you use.

1 How far is a shadow price good for? [5]

The maximum of $Z = 5x + 4y$ subject to $x + y \le b$, $2x + y \le 8$, $x, y \ge 0$ is 23 at (3, 2) when $b = 5$, and the final tableau shows a shadow price of 3 for the first constraint.

a. Kushal says, "With $b = 10$, the maximum will be $23 + 5 \times 3 = 38$." Find the true maximum for $b = 10$, by the simplex method or from the corners.

b. Solve $x + y = b$, $2x + y = 8$ for x and y in terms of b, and find the values of b for which that point is a corner of the region.

c. Explain where Kushal went wrong.

2 Changing a price [4]

Keep $x + y \le 5$, $2x + y \le 8$, $x, y \ge 0$, and maximize $Z = cx + 4y$.

a. For which values of c is (3, 2) still the best corner? Answer by comparing the slope of the objective with the slopes of the two constraints.

b. For $c = 4$, the final tableau's bottom row is (0, 0, 4, 0 | 20) under x, y, s_1, s_2. What does the 0 under the non-basic s_2 tell you? Find another best plan.

3 Three products at the pottery [6]

A pottery in Bhaktapur makes cups, bowls and pots. A cup takes 1 kg of clay and 2 units of painting time, a bowl 2 kg and 1 unit, a pot 1 kg and 3 units. Each day there are 40 kg of clay and 60 units of painting time. The profit is Rs 20 on a cup, Rs 30 on a bowl and Rs 25 on a pot.

a. Write the problem as a linear programming problem, with x cups, y bowls and z pots.

b. Solve it by the simplex method.

c. The cup is not made at all. Use the final tableau to say how much its profit would have to rise before making cups is worth it.

4 A minimum by the same tableau [5]

Minimize $C = x - 3y$ subject to $x + y \le 6$, $-x + 2y \le 6$, $x, y \ge 0$.

a. Explain why this is the same as maximizing $Z = -x + 3y$, and solve that by the simplex method.

b. Lhakpa says, "x only adds to C, so the best plan has $x = 0$." Find the best plan with $x = 0$, and compare.