Unit 16 · Linear Programming Problems

ANSWER KEY · FOR TEACHERS

Full marks 24

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 3

1. slack
2. tableau
3. pivot column
4. pivot row
5. corner
6. optimal

Each carries 0.5 mark.

2 True or false

U · 3

i. ✓ **True**

ii. ✗ **False**: It holds their negatives, -3 and -5, from $Z - 3x - 5y = 0$.

iii. ✗ **False**: The smallest ratio, over positive entries only: it is the first constraint to run out as the entering variable grows.

iv. ✓ **True**

v. ✓ **True**

vi. ✗ **False**: The slacks are basic at the start (the corner O, where $x = y = 0$).

Each carries 0.5 mark.

3 Multiple choice

i. **b.** 4 units of the resource were left unused — K · 1

$s_1 = 12 - (3x + 2y)$: the part of 12 not used. 4 unused means 8 used; a broken constraint would need $s_1 < 0$, never allowed.

ii. **b.** y — U · 1

The most negative entry, -5, is under y: raising y raises Z fastest. Negative entries remain, so it isn't optimal.

iii. **c.** the third (ratio 3) — U · 1

Ratios over positive entries only: $\frac{12}{3} = 4$ and $\frac{6}{2} = 3$; the smallest is 3. The -1 gives no ratio: that constraint never runs out as the variable grows.

iv. **c.** 23, at (3, 2) — A · 1

Z is the bottom right entry, 23; read the basic variables from their own rows: $y = 2$ and $x = 3$, so the point $(x, y) = (3, 2)$.

v. **c.** Z has no maximum (the problem is unbounded) — HA · 1

x should enter (-5), but its column has no positive entry: no constraint stops x growing, and Z grows without end. A negative entry is never a pivot.

4 Short answers

a. MARKING SCHEME — U · 2

✓ The ratios 8 and 10 as the values where each slack reaches 0.

✓ Past the smaller one a slack goes negative (outside the region).

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b. **MARKING SCHEME** **A · 3**

✓ The 1 under s_1 in the s_2 row should be 0.

✓ The bottom row should be $-4, -3$.

✓ The correct tableau.

c. **ANSWER** **A · 5**

$x + y + s_1 = 5$, $2x + y + s_2 = 8$, $Z - 5x - 4y = 0$.

Basic	x	y	s_1	s_2	RHS	Ratio
s_1	1	1	1	0	5	5
s_2	**2**	1	0	1	8	4
Z	-5	-4	0	0	0	

x enters (-5), s_2 leaves $(4 < 5)$. $R_2 \to \frac{1}{2}R_2$, $R_1 \to R_1 - R_2$, $R_3 \to R_3 + 5R_2$:

Basic	x	y	s_1	s_2	RHS	Ratio
s_1	0	$\mathbf{\frac{1}{2}}$	1	$-\frac{1}{2}$	1	2
x	1	$\frac{1}{2}$	0	$\frac{1}{2}$	4	8
Z	0	$-\frac{3}{2}$	0	$\frac{5}{2}$	20	

y enters, s_1 leaves $(2 < 8)$. $R_1 \to 2R_1$, $R_2 \to R_2 - \frac{1}{2}R_1$, $R_3 \to R_3 + \frac{3}{2}R_1$:

Basic	x	y	s_1	s_2	RHS
y	0	1	2	-1	2
x	1	0	-1	1	3
Z	0	0	3	1	23

No negative entry: **maximum $Z = 23$ at $x = 3$, $y = 2$.** Check: $15 + 8 = 23$, $3 + 2 = 5$ ✓, $6 + 2 = 8$ ✓.

MARKING SCHEME

✓ Slacks and the first tableau with $-5, -4$.

✓ x enters, s_2 leaves (ratios 5 and 4).

✓ Second tableau, $Z = 20$ at $(4, 0)$.

✓ y enters, s_1 leaves; the final tableau.

✓ Maximum 23 at $(3, 2)$.

5 Think

ANSWER **HA · 3**

$x + y = 6$, $2x + y = 8$: $x = 2$, $y = 4$, $Z = 10 + 16 = 26 = 23 + 3$ ✓ (and $(2, 4)$ is feasible). She is right. The entries under the slacks are the **shadow prices**: one more unit of the first resource is worth 3 in Z, one more of the second only 1 ($x + y = 5$, $2x + y = 9$ gives $(4, 1)$, $Z = 24$). So the first resource is the one to buy more of.

MARKING SCHEME

✓ $(2, 4)$, $Z = 26$, 3 more.

✓ The 3 and the 1 are what one more unit of each resource adds to Z.

✓ So the first resource is worth more (with the second checked, or reasoned).

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6 Activity: Corners by hand, corners by tableau

(not marked)

WHAT STUDENTS SHOULD FIND

Four corners: $O(0, 0)$, $Z = 0$; $(4, 0)$, $Z = 20$; $(3, 2)$, $Z = 23$; $(0, 5)$, $Z = 20$. The tableaux go $O \to (4, 0) \to (3, 2)$ along the edges, Z rising each time; $(0, 5)$ is never visited. On a big problem with thousands of corners, that skipping is what makes the method fast.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	4	7	9	4	**24**

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