Unit 15 · System of Linear Equations

WORKSHEET

Full marks 24

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 24

1 Key words [3]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

back substitution	determinant	diagonally dominant	iterative	Jacobi	pivot	swapped	upper triangular

Gauss elimination uses row operations to turn the augmented matrix into _____ form, and then finds the unknowns by _____, the last one first. The entry used to clear the entries below it is the _____; if it is 0, the rows are _____. The Gauss–Seidel method is an _____ method; it is sure to converge when the system is _____.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. Adding a multiple of one row to another never changes the solutions of a system. ☐

ii. A last row (0 0 0 | 0) means the system has no solution. ☐

iii. In the Gauss–Seidel method, every new value is worked out from the values of the previous iteration only. ☐

iv. Reordering the equations can make a system diagonally dominant. ☐

v. Gauss elimination gives the exact solution; Gauss–Seidel gives it to a chosen accuracy. ☐

vi. A system that is not diagonally dominant can never converge under Gauss–Seidel. ☐

3 Multiple choice [5]

Choose the correct option for each of the following questions.

i. In Gauss elimination, after the forward elimination the coefficient matrix is: [1]

a. diagonal **b.** upper triangular
c. lower triangular **d.** the identity

ii. Which system is diagonally dominant as it is written? [1]

a. $x + 5y = 7,\ 4x - y = 2$ **b.** $4x - y = 2,\ x + 5y = 7$
c. $2x + 3y = 5,\ x + y = 2$ **d.** $x - 4y = -7,\ 5x + y = 7$

iii. After elimination, the last row of a system's augmented matrix is (0 0 0 | 5). The system has: [1]

a. no solution **b.** exactly one solution
c. infinitely many solutions **d.** the solution $z = 5$

iv. One iteration of Gauss–Seidel for $4x + y = 9$, $x + 3y = 5$, starting from $(0, 0)$, gives: **[1]**

a. $(2.25, 1.667)$ **b.** $(2.25, 0.917)$

c. $(2, 1)$ **d.** $(2.25, 0)$

v. For which value of k does the system $x + y + z = 3$, $x + 2y + 3z = 6$, $2x + 3y + kz = 9$ have infinitely many solutions? **[1]**

a. no value of k **b.** $k = 3$

c. $k = 4$ **d.** every value except 4

4 Short answers **[10]**

Write short answers to the following questions.

a. Before using Gauss–Seidel on $x - 4y = -7$, $5x + y = 7$, Anuska swaps the two equations. Why? **[2]**

b. Do two iterations of the Gauss–Seidel method for $10x + y + z = 12$, $x + 10y + z = 12$, $x + y + 10z = 12$, starting from $(0, 0, 0)$. Give the values to four decimal places. **[3]**

c. Solve by Gauss elimination: $x + y + z = 6$, $2x - y + z = 3$, $x + 2y - z = 2$. **[5]**

Answer:

5 Think [3]

Bishal uses Gauss–Seidel on $x - 4y = -7$, $5x + y = 7$ as they are written, taking $x = 4y - 7$ from the first and $y = 7 - 5x$ from the second, starting from $(0, 0)$. Work three iterations, say what happens, and explain why.

6 Activity: Newest values or old ones? (not marked)

You need: a calculator. Solve $4x + y = 9$, $x + 3y = 5$ from $(0, 0)$ twice: by Gauss–Seidel (use the new x at once to find y), and by Jacobi's method (find both from the old values).

Iteration	GS x	GS y	Jacobi x	Jacobi y
1				
2				
3				
4				
5				

After how many iterations does each method give $(2, 1)$ to three decimal places?