Unit 14 · Differential Equations

WORKSHEET

Full marks 35

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 35

1 Key words [4]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

arbitrary	**degree**	**exact**	**general**	**homogeneous**	**implicit**	**integrating factor**	**order**
particular	**separable**						

The highest derivative in a differential equation gives its _____; the power of that derivative, once roots are cleared, is its _____. A solution with as many _____ constants as the order is the _____ solution, and a condition such as $y = 1$ at $x = 0$ picks out a _____ solution. An equation that can be written $g(y)\,dy = f(x)\,dx$ is _____; one of the form $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ is _____. To solve $\frac{dy}{dx} + Py = Q$, multiply by the _____ $e^{\int P\,dx}$.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = x$ has order 2 and degree 1. ☐

ii. The degree of $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is 1. ☐

iii. Dividing $\frac{dy}{dx} = ky$ by y can lose the solution $y = 0$. ☐

iv. The integrating factor of $x\frac{dy}{dx} - 2y = x^3$ is $e^{\int 2\,dx}$. ☐

v. The family $y = Ax + B$ gives a differential equation of order 2. ☐

vi. Every equation of the first order and first degree is separable. ☐

3 Multiple choice [5]

Choose the correct option for each of the following questions.

i. The order and degree of $\left(\frac{d^2y}{dx^2}\right)^2 + \frac{dy}{dx} = 0$ are: [1]

a. 1 and 2 **b.** 2 and 2
c. 2 and 1 **d.** 1 and 1

ii. The integrating factor of $\frac{dy}{dx} + \frac{2}{x}y = x$, for $x > 0$, is: [1]

a. e^{2x} **b.** x^2
c. $\frac{1}{x^2}$ **d.** $2\ln x$

iii. The general solution of $\frac{dy}{dx} = \frac{y}{x}$ $(x \neq 0)$ is: [1]

a. $y = Cx$ **b.** $y = x + C$

c. $xy = C$ **d.** $y = Ce^x$

iv. The general solution of $\frac{dy}{dx} = e^{x-y}$ is: [1]

a. $e^{-y} = e^x + C$ **b.** $e^y = e^x + C$

c. $y = x + C$ **d.** $e^{x+y} = C$

v. $y = Ce^x - x - 1$ is the general solution of $\frac{dy}{dx} = x + y$. Which of its solutions is a straight line? [1]

a. $y = -x - 1$ **b.** $y = x + 1$

c. $y = -x$ **d.** none: every solution has e^x in it

4 Short answers [12]

Write short answers to the following questions.

a. Form the differential equation whose general solution is $y = A\cos x + B\sin x$. [2]

b. Solving $\frac{dy}{dx} = y^2$, Swapnil divides by y^2 and gets $y = -\frac{1}{x + C}$. Show that $y = 0$ is also a solution, and explain why his family misses it. [2]

c. Solve $\frac{dy}{dx} = \frac{1 + y^2}{1 + x^2}$, and find the solution with $y = 1$ when $x = 0$. [3]

d. Solve $x\frac{dy}{dx} + 2y = x^2$, $x > 0$, and find the solution through $(1, 1)$. **[5]**

Answer: _____

5 Long answer **[8]**

Answer the following question. Show your working.

a. Solve $x\frac{dy}{dx} = y + \sqrt{x^2 + y^2}$, for $x > 0$. [4]

b. A cup of tea at 90 °C is left in a room at 20 °C, and cools by Newton's law, $\frac{dT}{dt} = -k(T - 20)$. After 10 minutes it is at 60 °C. Solve the equation, and find when the tea will be at 40 °C. [4]

Science Playground, Katunje, Bhaktapur
https://science-playground.com/learn/grade-12-math/differential-equations

6 Think [3]

Sujan solves $\frac{dy}{dx} = 2\sqrt{y}, y \geq 0$, and gets $y = (x + C)^2$ for $x + C \geq 0$. Show that $y = 0$ is also a solution, and that two different solutions pass through $(0, 0)$. What does this tell you about dividing by $\sqrt{y}$?

7 Activity: A cooling cup (not marked)

You need: a thermometer (a kitchen or laboratory one), a cup of hot water, and a clock. Note the room's temperature T_s, then the water's temperature every 2 minutes for 16 minutes.

t (min)	T (°C)	$T - T_s$	$\ln(T - T_s)$
0			
2			
4			
6			
8			
10			
12			
14			
16			

Plot $\ln(T - T_s)$ against t. Is it close to a straight line? What is its slope?