Unit 13 · Anti-derivatives

WORKSHEET

Full marks 34

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 34

1 Key words [3]

Match each integral in Column A with its anti-derivative or method in Column B. Write the letter in the box. One item in Column B is not needed. Each match carries 0.5 mark.

Column A

1. $\int \frac{dx}{x^2 + a^2}$
2. $\int \frac{dx}{\sqrt{a^2 - x^2}}$
3. $\int \frac{dx}{x^2 - a^2}$
4. $\int \frac{dx}{\sqrt{x^2 + a^2}}$
5. $\int \frac{dx}{x\sqrt{x^2 - a^2}}, x > a$
6. $\int \frac{dx}{a + b\cos x}$

Column B

a. the substitution $t = \tan \frac{x}{2}$

b. $\ln \left| x + \sqrt{x^2 + a^2} \right| + C$

c. $\frac{1}{a} \tan^{-1} \frac{a}{x} + C$

d. $\frac{1}{a} \tan^{-1} \frac{x}{a} + C$

e. $\sin^{-1} \frac{x}{a} + C$

f. $\frac{1}{2a} \ln \left| \frac{x - a}{x + a} \right| + C$

g. $\frac{1}{a} \sec^{-1} \frac{x}{a} + C$

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. For a factor $(x - a)^2$ in the bottom, the partial fractions are $\frac{A}{x - a} + \frac{B}{(x - a)^2}$. ☐

ii. $\frac{x^3}{x^2 - 1}$ can be put into partial fractions straight away. ☐

iii. $\sin^{-1} \frac{x}{a} + C$ and $-\cos^{-1} \frac{x}{a} + C$ are both right answers for $\int \frac{dx}{\sqrt{a^2 - x^2}}$. ☐

iv. $\int \frac{dx}{x^2 + 4} = \tan^{-1} \frac{x}{2} + C$. ☐

v. $x^2 + 2x + 5 = (x + 1)^2 + 4$, so $\int \frac{dx}{x^2 + 2x + 5}$ is a $\tan^{-1}$ form. ☐

vi. An answer found with $t = \tan \frac{x}{2}$ can be used for a definite integral from 0 to 2π. ☐

3 Multiple choice [5]

Choose the correct option for each of the following questions.

i. $\int \frac{dx}{9+x^2}$ is: [1]

a. $\tan^{-1}\frac{x}{3}+C$

b. $\frac{1}{3}\tan^{-1}\frac{x}{3}+C$

c. $3\tan^{-1}\frac{x}{3}+C$

d. $\frac{1}{6}\ln\left|\frac{x-3}{x+3}\right|+C$

ii. $\int \frac{dx}{x^2-6x+13}$ is: [1]

a. $\frac{1}{2}\tan^{-1}\frac{x-3}{2}+C$

b. $\frac{1}{2}\tan^{-1}\frac{x+3}{2}+C$

c. $\tan^{-1}\frac{x-3}{2}+C$

d. $\frac{1}{4}\ln\left|\frac{x-5}{x-1}\right|+C$

iii. $\int \frac{dx}{4x^2+9}$ is: [1]

a. $\frac{1}{3}\tan^{-1}\frac{2x}{3}+C$

b. $\frac{1}{2}\tan^{-1}\frac{2x}{3}+C$

c. $\frac{1}{6}\tan^{-1}\frac{2x}{3}+C$

d. $\frac{1}{12}\tan^{-1}\frac{x}{3}+C$

iv. $\int \frac{dx}{x(x+1)}$ is: [1]

a. $\ln\left|\frac{x}{x+1}\right|+C$

b. $\ln|x(x+1)|+C$

c. $\ln\left|\frac{x+1}{x}\right|+C$

d. $\tan^{-1}(2x+1)+C$

v. $\int \frac{dx}{x^2-4x+3}$ is: [1]

a. $\frac{1}{2}\ln\left|\frac{x-1}{x-3}\right|+C$

b. $\frac{1}{2}\ln\left|\frac{x-3}{x-1}\right|+C$

c. $\tan^{-1}(x-2)+C$

d. $\ln|(x-1)(x-3)|+C$

4 Short answers [12]

Write short answers to the following questions.

a. Explain why $\frac{x^3}{x^2-1}$ must be divided before it is split into partial fractions, and write the result of the division. [2]

b. Integrate $\int \frac{dx}{x^2+2x+5}$. [2]

c. Soyana wrote: $\int \frac{dx}{4x^2+9} = \frac{1}{3}\tan^{-1}\frac{x}{3} + C$. Check her answer by differentiating it, find her mistake, and write the correct answer. [3]

d. Integrate $\int \frac{3x+5}{x^2+2x+5}\,dx$. [5]

Answer:

5 Long answer [8]

Answer the following question. Show your working.

a. Integrate $\int \frac{x+5}{(x-1)(x+2)^2}\,dx$. [4]

b. Using $t = \tan\frac{x}{2}$, integrate $\int \frac{dx}{5+4\cos x}$. [3]

c. Hence evaluate $\int_0^{\pi/2} \frac{dx}{5+4\cos x}$. [1]

6 Think [3]

Manish works out $\int_{-1}^{1} \frac{dx}{x^2} = \left[-\frac{1}{x}\right]_{-1}^{1} = -1 - 1 = -2$. Without integrating anything else, explain why his answer can't be right, and what went wrong.

7 Activity: A dip chart for a bottle

(not marked)

You need: a straight-sided round plastic bottle, a measuring cup and a ruler. Lay the bottle on its side, measure its inside diameter, pour in water 100 mL at a time and measure the depth each time.

Water poured (mL)	Depth (cm)	Share of the diameter	Share of the full volume

At a quarter of the depth, what share of the volume is in the bottle? Compare it with the 19.6% the unit found for the diesel tank.