Unit 12 · Derivatives

WORKSHEET

Full marks 34

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 34

1 Key words [3]

Match each item in Column A with Column B. Write the letter in the box. One item in Column B is not needed. Each match carries 0.5 mark.

Column A		Column B
1. $\sinh x$	☐	**a.** $\dfrac{e^x + e^{-x}}{2}$
2. $\cosh x$	☐	**b.** $\dfrac{0}{0}$ and $\dfrac{\infty}{\infty}$
3. $\tanh^{-1} x$, for $\lvert x\rvert < 1$	☐	**c.** $\ln\left(x + \sqrt{x^2 - 1}\right)$
4. the differential dy	☐	**d.** $\dfrac{1}{2}\ln\dfrac{1+x}{1-x}$
5. the slope of the normal where the tangent's slope is m	☐	**e.** $f'(x)\,dx$
6. the forms L'Hospital's rule works on	☐	**f.** $\dfrac{e^x - e^{-x}}{2}$
		g. $-\dfrac{1}{m}$

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. $\dfrac{d}{dx}(\cosh x) = -\sinh x$. ☐

ii. $\cosh x \geq 1$ for every real x. ☐

iii. L'Hospital's rule differentiates the quotient $\dfrac{f(x)}{g(x)}$ by the quotient rule. ☐

iv. The differential dy is always exactly equal to the change Δy. ☐

v. If the side of a square has a 1% error, its area has about a 2% error. ☐

vi. $\tanh^{-1} x$ is defined for every real x. ☐

3 Multiple choice [5]

Choose the correct option for each of the following questions.

i. $\dfrac{d}{dx}(\sinh x)$ is: [1]

a. $\cosh x$ **b.** $-\cosh x$

c. $\sinh x$ **d.** $\operatorname{sech}^2 x$

ii. $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$ is: [1]

a. 0

b. $\frac{1}{2}$

c. 1

d. 2

iii. $\frac{d}{dx}\left[\cosh^{-1}(2x)\right]$, for $x > \frac{1}{2}$, is: [1]

a. $\frac{1}{\sqrt{4x^2 - 1}}$

b. $\frac{2}{\sqrt{4x^2 - 1}}$

c. $\frac{2}{\sqrt{4x^2 + 1}}$

d. $\frac{2}{\sqrt{1 - 4x^2}}$

iv. The tangent to $y = x^2 - 4x$ at $(1, -3)$ is: [1]

a. $y = 2x - 5$

b. $y = -2x - 1$

c. $y = -2x + 1$

d. $2y = x - 7$

v. The tangent to $y = e^x$ at one point passes through the origin. That point is: [1]

a. $(0, 1)$

b. $(1, e)$

c. $(-1, e^{-1})$

d. (e, e^e)

4 Short answers [12]

Write short answers to the following questions.

a. Can L'Hospital's rule be used for $\lim_{x \to 0} \frac{\sin x}{x + 1}$? Give a reason, and find the limit. [2]

b. Using differentials, find an approximate value of $\sqrt{50}$. [2]

c. Aaryan wrote: $\lim_{x \to 0} \frac{e^x - 1}{x^2} = \lim_{x \to 0} \frac{e^x}{2x} = \lim_{x \to 0} \frac{e^x}{2} = \frac{1}{2}$. Find his mistake, and say what is true about the limit. [3]

d. Show that (1, 2) lies on the curve $x^2 + xy + y^2 = 7$, and find the equations of the tangent and the normal to the curve there. **[5]**



Answer: _____

5 Long answer **[8]**

Answer the following question. Show your working.

a. Starting from $\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$, show that $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2 + 1}}$. [3]

b. If $y = (\sinh^{-1} x)^2$, prove that $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$. [3]

c. Water runs into a tank shaped like a cone, vertex down, 4 m deep and 2 m in radius at the top, at 0.5 m^3 a minute. How fast is the water level rising when the water is 1 m deep? [2]

6 Think [3]

The side of a square grows from x to $x + dx$. Kushal says the differential $dA = 2x\,dx$ is "the area of two thin strips". Draw the two squares, one inside the other, and use the drawing to explain what dA is, what the true change ΔA is, and what dA leaves out.

7 Activity: How good is dy? (not marked)

You need: a calculator. For $y = x^3$ at $x = 2$, $\frac{dy}{dx} = 12$. Work out the true change $\Delta y = (2 + dx)^3 - 8$ and the differential $dy = 12\,dx$ for each dx.

dx	Δy	dy	$\Delta y - dy$
0.5			
0.1			
0.01			

What happens to the difference $\Delta y - dy$ each time dx is made 10 times smaller?