Unit 12 · Derivatives

HARDER PRACTICE QUESTIONS

Full marks 29

Write your answers on separate paper. Show your working. Angles are in radians unless a question says otherwise.

1 Two constants to choose [4]

$\lim_{x \to 0} \dfrac{a\cosh x + b\cos x - 2}{x^4}$ is a finite number for exactly one pair of constants a and b. Using L'Hospital's rule, find a, b and the limit. Say at each step why the rule may be used.

2 Tape or calipers? [5]

Prapti wants the volume of a football. She can measure its circumference with a tape, 70 cm to within ±0.5 cm, or its diameter with calipers to within ±0.2 cm.

a. Using differentials, find the percentage error in the volume worked out from the circumference.

b. Find the diameter the ball has, and the percentage error in the volume worked out from the diameter.

c. Bishal says the calipers must be better, "because 0.2 cm is less than 0.5 cm". Is he right? Explain.

3 Lines through a point off the curve [6]

a. Find the equations of the two tangents to $y = x^2$ that pass through $(1, -3)$, and their points of contact.

b. Find every normal to $y = x^2$ that passes through $\left(0, \dfrac{5}{2}\right)$. (There are more than two.)

4 A drone going up [5]

Rabina stands 40 m from the point where a drone takes off. It rises straight up at a steady 3 m/s.

a. How fast is its angle of elevation from her increasing when it is 30 m up? Give the answer in radians a second.

b. How fast is its distance from her increasing at that moment?

c. Her friend writes, "At that moment the distance is $\sqrt{40^2 + 30^2} = 50$ m, and the rate of change of 50 is 0." Explain the mistake.

d. At what height is the angle of elevation increasing fastest? Explain why.

5 One derivative, two functions [4]

Let $y = \tanh^{-1}\dfrac{2x}{1+x^2}$, for $x \neq \pm 1$.

a. Show that $\dfrac{dy}{dx} = \dfrac{2}{1-x^2}$.

b. Pratik says, "So $y = 2\tanh^{-1} x$." Explain why that can't be right for $|x| > 1$, and say what y is there.

c. Why is $x = 1$ left out?

6 A hanging cable [5]

A power cable hangs between two poles 100 m apart in the shape $y = a\cosh\frac{x}{a}$ (meters), with its lowest point at $x = 0$ and $a = 200$.

a. Show that $\frac{d^2y}{dx^2} = \frac{1}{a}\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$.

b. Find the sag, how far the cable's ends are above its lowest point.

c. Dipesh tries to find the sag with a differential from the lowest point: $dy = \sinh(0) \times 50 = 0$. Explain why the differential is no help here.