Unit 11 · Probability

HARDER PRACTICE QUESTIONS

Full marks 23

Write your answers on separate paper. Show your working. Give probabilities as fractions in their lowest terms, or as decimals to three places.

1 How many bulbs to test? [6]

A shop receives LED bulbs in boxes of 20. The owner tests n bulbs from each box, chosen at random (without putting back), and accepts the box only if none of them is faulty.

a. She wants a box with 4 faulty bulbs to be accepted with a probability of less than 0.2. Find the least n that does this.

b. Her assistant Bibek works it out as if the draws were independent, each bulb faulty with probability $\frac{4}{20}$. What n does his method give, and why is it wrong here?

c. With your n from a, find the probability that a box with only 1 faulty bulb is accepted, and say what this means for the owner.

2 What a positive test means [5]

A health camp screens 5,000 students for a condition that 2% of them have. The test is positive for 90% of the students who have the condition, and also, wrongly, for 5% of those who don't.

a. Copy and complete a table of the numbers of students: has the condition or not, against positive or negative.

b. Find the probability that a student chosen at random tests positive.

c. Rojina tests positive. Find the probability that she has the condition.

d. Rojina's friend says, "The test is right 90% of the time, so she is 90% sure to have it." Explain the mistake.

3 Independent is not the same as separate [4]

a. Show that if A and B are independent, then $\overline{A}$ and $\overline{B}$ are independent too.

b. Show that two mutually exclusive events, each with a probability more than 0, can never be independent.

c. A die is thrown. A: an even number; B: a number up to 4. Are A and B independent? Use a. to find the probability of an odd number more than 4, and check it by listing.

4 The third egg [5]

A tray has 12 eggs, 3 of them cracked. Eggs are taken one at a time, without putting back.

a. Find the probability that the **third** egg taken is cracked, by working through the first two draws.

b. Find the probability that the third egg is the **first** cracked egg taken.

c. Given that exactly one of the first three eggs is cracked, find the probability that it is the third.

5 Make your own [3]

For one throw of a die, B is "an even number". Make up three events A (each a set of faces) so that $P(B \mid A)$ is more than, less than and equal to $P(B)$. Show the working for each, and say which of your A is independent of B.