Unit 11 · Probability

ANSWER KEY · FOR TEACHERS

Full marks 27

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 4

1. dependent
2. conditional
3. given
4. multiplication
5. independent
6. combinations
7. none
8. complement

Each carries 0.5 mark.

2 True or false

U · 3

i. ✗ **False**: They have the same top, $P(A \cap B)$, but divide by different numbers, $P(A)$ and $P(B)$; they are equal only when $P(A) = P(B)$.

ii. ✓ **True**

iii. ✗ **False**: That is the multiplication law, true for any two events; they are independent only when $P(B \mid A) = P(B)$.

iv. ✓ **True**

v. ✓ **True**

vi. ✗ **False**: Knowing that one happened makes the other impossible, which changes its chance: they are dependent (if both chances are more than 0).

Each carries 0.5 mark.

3 Multiple choice

i. **a.** 0.15 — K · 1

Multiplication law: $0.3 \times 0.5 = 0.15$. 0.6 divides $0.3 \div 0.5$; 0.8 adds.

ii. **b.** $\frac{1}{221}$ — U · 1

$\frac{4}{52} \times \frac{3}{51} = \frac{12}{2652} = \frac{1}{221}$. $\frac{1}{169} = \left(\frac{4}{52}\right)^2$ puts the first card back.

iii. **b.** 0.4 — U · 1

Independent: knowing B doesn't change A's chance, so $P(A) = P(A \mid B) = 0.4$.

iv. **c.** $\frac{23}{28}$ — A · 1

None from Management: $\frac{C(5,3)}{C(8,3)} = \frac{10}{56} = \frac{5}{28}$, so at least one: $1 - \frac{5}{28} = \frac{23}{28}$. $\frac{5}{28}$ is the complement itself; $\frac{15}{28}$ is exactly one.

v. **b.** $P(B \mid A) = 0.4$, and A and B are independent — HA · 1

$P(A \cap B) = P(B)P(A \mid B) = 0.4 \times 0.5 = 0.2$, so $P(B \mid A) = \frac{0.2}{0.5} = 0.4 = P(B)$: independent. They can't be mutually exclusive, since $P(A \cap B) = 0.2 \neq 0$.

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4 Short answers

a. MARKING SCHEME A · 2

✓ $P(F \mid B) = \frac{12}{25}$.

✓ $P(B \mid F) = \frac{4}{5}$.

b. MARKING SCHEME U · 2

✓ After a cracked egg the chance is $\frac{2}{11}$; after a whole one, $\frac{3}{11}$.

✓ The second draw's chance depends on the first, so they are dependent.

c. MARKING SCHEME A · 3

✓ The two orders, red then white and white then red.

✓ $\frac{8}{15}$.

✓ $\frac{2}{3}$.

5 Long answer

ANSWER A · 5

$n(S) = C(12, 4) = 495$.

a. $\frac{C(5, 2) \times C(7, 2)}{495} = \frac{10 \times 21}{495} = \frac{210}{495} = \frac{14}{33}$.

b. No girls: $C(7, 4) = 35$, so $P = 1 - \frac{35}{495} = \frac{460}{495} = \frac{92}{99}$.

c. "Exactly 2 girls" lies inside "at least one girl", so $P(\text{exactly 2} \mid \text{at least 1}) = \frac{210}{460} = \frac{21}{46}$.

What it means: knowing the team isn't all boys raises the chance of exactly two girls only a little, from about 0.42 to 0.46, because all-boy teams were rare.

MARKING SCHEME

✓ $n(S) = C(12, 4) = 495$ and $C(5, 2) \times C(7, 2) = 210$.

✓ $\frac{14}{33}$.

✓ $1 - \frac{35}{495} = \frac{92}{99}$.

✓ The intersection of "exactly 2" and "at least 1" is "exactly 2".

✓ $\frac{210}{460} = \frac{21}{46}$.

6 Think

ANSWER HA · 3

Neither: every turn has the same chance, $\frac{1}{30}$.

The second person wins only if the first did not: $\frac{29}{30} \times \frac{1}{29} = \frac{1}{30}$. In the same way the last person wins only if all 29 before missed, $\frac{29}{30} \cdot \frac{28}{29} \cdots \frac{1}{2} \times 1 = \frac{1}{30}$. Drawing later gives a better chance **if** the prize is still there, but a chance that it is already gone, and the two balance exactly.

MARKING SCHEME

✓ Neither: every turn has the chance $\frac{1}{30}$.

✓ Second: $\frac{29}{30} \times \frac{1}{29} = \frac{1}{30}$.

✓ A later turn is better only if the prize is still there, which balances the chance that it has gone.

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7 Activity: Knowing changes the chance

(not marked)

WHAT STUDENTS SHOULD FIND

About a sixth of all throws (12 of 72) have a sum of 10 or more, but among the throws with a 6 (about 22) the share is much higher, near $\frac{5}{11}$: of the 11 outcomes with a 6, five, (6, 4), (6, 5), (6, 6), (4, 6) and (5, 6), add to 10 or more.

Knowing there is a 6 changes the chance: the events are dependent. With only 72 throws the fractions wander; the class's throws together come closer.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	5	7	11	4	**27**

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