Unit 9 · Product of Vectors

WORKSHEET

Full marks 37

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 37

1 Key words [4]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

directrix	**eccentricity**	**parallel**	**parallelogram**	**perpendicular**	**projection**	**right-hand**
scalar	**vector**	**work**				

The product $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$ is a number, so it is called the _____ product. It is 0 when the vectors are _____. Divided by $|\vec{a}|$, it gives the _____ of $\vec{b}$ on $\vec{a}$. The _____ done by a force is the scalar product of the force and the displacement. The _____ product $\vec{a} \times \vec{b}$ is at right angles to both $\vec{a}$ and $\vec{b}$, and it is $\vec{0}$ when they are _____. Its length is the area of the _____ on $\vec{a}$ and $\vec{b}$, and its direction is given by the _____ rule.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$. ☐

ii. $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$. ☐

iii. $\vec{i} \times \vec{i} = \vec{0}$. ☐

iv. If $\vec{a} \cdot \vec{b} = 0$ for two non-zero vectors, they are parallel. ☐

v. $|\vec{a} \times \vec{b}|$ is the area of the triangle on $\vec{a}$ and $\vec{b}$. ☐

vi. $\vec{a} \cdot \vec{a} = |\vec{a}|^2$. ☐

3 Multiple choice [5]

Choose the correct option for each of the following questions.

i. $\vec{j} \times \vec{k}$ is: [1]

a. $\vec{i}$ **b.** $-\vec{i}$

c. 0 **d.** 1

ii. $(2\vec{i} - \vec{j} + 3\vec{k}) \cdot (\vec{i} + 4\vec{j} + 2\vec{k})$ is: [1]

a. 4 **b.** 12

c. $2\vec{i} - 4\vec{j} + 6\vec{k}$ **d.** -4

iii. The angle between $\vec{i} + \vec{j}$ and $\vec{j} + \vec{k}$ is: [1]

a. 30° **b.** 45°

c. 60° **d.** 90°

iv. $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $\vec{a} \cdot \vec{b} = 6$. Then $|\vec{a} \times \vec{b}|$ is: [1]

a. 6 **b.** $6\sqrt{3}$

c. 12 **d.** $2\sqrt{3}$

v. For two non-zero vectors, $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$. Then: [1]

a. $\vec{a}$ and $\vec{b}$ are parallel **b.** $\vec{a}$ and $\vec{b}$ are perpendicular

c. $|\vec{a}| = |\vec{b}|$ **d.** $\vec{a} = \vec{b}$

4 Short answers [9]

Write short answers to the following questions.

a. Explain why $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$ have the same length but point opposite ways. [2]

b. For $\vec{a} = \vec{i} + 2\vec{j} + 2\vec{k}$ and $\vec{b} = 3\vec{i} + 4\vec{k}$, Bikash writes the projection of $\vec{b}$ on $\vec{a}$ as $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{11}{5}$. Find his mistake and the right value. [2]

c. A force $\vec{F} = 2\vec{i} + 3\vec{j} + \vec{k}$ newtons moves a body from (1, 0, 2) to (4, 2, 3), in meters. Find the work done. [2]

d. Find the unit vectors perpendicular to both $\vec{a} = \vec{i} + \vec{j}$ and $\vec{b} = \vec{j} + \vec{k}$. [3]

5 Long answers [13]

Write long answers to the following questions. Show your working.

a. Prove by vectors that the angle in a semicircle is a right angle. **[5]**

b. The points $A(1, 2, 0)$, $B(3, 3, 2)$ and $C(2, 4, -1)$ are the corners of a triangle. **[8]**

i. Find $\overrightarrow{AB}$, $\overrightarrow{AC}$ and their lengths.

ii. Find the angle A.

iii. Find $\overrightarrow{AB} \times \overrightarrow{AC}$ and the area of the triangle.

iv. Find the length of the perpendicular from C to AB.

Answer:

6 Think [3]

$\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Show that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$, and say what this means for the triangle whose sides are $\vec{a}, \vec{b}, \vec{c}$.

7 Activity: The area of a field from its corners (not marked)

On squared paper, mark $A(0, 0)$, $B(6, 1)$, $C(7, 5)$ and $D(2, 6)$, and join them in order. Cut the field into the triangles ABC and ACD. Find each area as half of $x_1y_2 - x_2y_1$ for the position vectors of its two corners other than A, then count squares to check.

Triangle	Vectors	$x_1y_2 - x_2y_1$	Area
ABC	(6, 1), (7, 5)		
ACD	(7, 5), (2, 6)		
field			

How close is your count of the squares?