Unit 8 · Conic Section

WORKSHEET

Full marks 37

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 37

1 Key words [4]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

directrix	eccentricity	ellipse	focus	hyperbola	latus rectum	normal	pivot	tangent	telescope

A line that meets a circle at exactly one point is a _____; the line through that point at right angles to it is the _____. A conic is the path of a point whose distance from a fixed point, the _____, is a fixed multiple of its distance from a fixed line, the _____. That multiple is the _____ e: $e = 1$ gives a parabola, $e < 1$ an _____ and $e > 1$ a _____. The chord through a focus at right angles to the axis is the _____.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. $y = mx + c$ touches $x^2 + y^2 = a^2$ when $c^2 = a^2(1 + m^2)$. ☐

ii. The eccentricity of a parabola is more than 1. ☐

iii. For every point of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$, the distances to the two foci add up to $2a$. ☐

iv. For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $b^2 = a^2(1 - e^2)$. ☐

v. If ${x_1}^2 + {y_1}^2 + 2gx_1 + 2fy_1 + c < 0$, the point (x_1, y_1) is outside the circle. ☐

vi. The normal at any point of a circle passes through its center. ☐

3 Multiple choice [5]

Choose the correct option for each of the following questions.

i. The focus of the parabola $y^2 = 12x$ is: [1]

a. $(12, 0)$ **b.** $(3, 0)$

c. $(0, 3)$ **d.** $(-3, 0)$

ii. The eccentricity of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ is: [1]

a. $\frac{3}{5}$ **b.** $\frac{4}{5}$

c. $\frac{5}{4}$ **d.** $\frac{9}{25}$

iii. The line $y = mx + 3$ touches the parabola $y^2 = 12x$ when m is: [1]

a. 1 **b.** 3

c. $\frac{1}{3}$ **d.** 4

iv. The length of the tangent from (4, 5) to the circle $x^2 + y^2 - 2x - 4y - 4 = 0$ is: [1]

a. 3 **b.** 9

c. $\sqrt{41}$ **d.** 5

v. A hyperbola has eccentricity $\sqrt{2}$. The angle between its asymptotes is: [1]

a. 45° **b.** 60°

c. 90° **d.** 120°

4 Short answers [9]

Write short answers to the following questions.

a. Explain why the tangent at a point P of a circle is at right angles to the radius to P. [2]

b. Rajesh says the foci of $\frac{x^2}{9} + \frac{y^2}{25} = 1$ are $(\pm 4, 0)$. Find his mistake and the right foci. [2]

c. Find the tangent and the normal to $x^2 + y^2 = 25$ at $(3, -4)$. [2]

d. Find the vertex, focus and directrix of the parabola $y^2 + 4y - 12x + 16 = 0$. [3]

5 Long answers [13]

Write long answers to the following questions. Show your working.

a. Find the equation of the hyperbola with vertices $(\pm 3, 0)$ and foci $(\pm 5, 0)$. Find its eccentricity, directrices, latus rectum and asymptotes. **[5]**

b. Halley's comet goes round the Sun on an ellipse with the Sun at a focus. At its nearest it is about 0.59 AU from the Sun, and at its farthest about 35.1 AU (1 AU is the Earth's average distance from the Sun; the numbers are rounded). **[8]**

i. Find the semi-major axis a.

ii. Find the distance ae from the ellipse's center to the Sun, and the eccentricity e.

iii. Find the semi-minor axis b.

iv. Write the equation of the orbit, with the center at the origin and the major axis along the X-axis.

v. Compare e with the Earth's orbit (0.0167). What does it say about the comet's path?

Answer:

6 Think [3]

Show that the line $lx + my + n = 0$ touches the parabola $y^2 = 4ax$ when $ln = am^2$ $(l \neq 0)$.

7 Activity: An ellipse with a string (not marked)

Push two pins into a board 8 cm apart, and tie the ends of a string 10 cm long to them. Hold the string tight with a pencil and draw round the pins, keeping it tight: the pencil's distances from the two pins always add up to 10 cm.

Quantity	Predicted	Measured
length of the major axis $2a$		
length of the minor axis $2b$		
eccentricity e		

What happens to the curve if the pins are moved closer together?