Unit 8 · Conic Section

ANSWER KEY · FOR TEACHERS

Full marks 37

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 4

1. tangent
2. normal
3. focus
4. directrix
5. eccentricity
6. ellipse
7. hyperbola
8. latus rectum

Each carries 0.5 mark.

2 True or false

U · 3

i. ✓ **True**

ii. ✗ **False**: It is exactly 1: every point is as far from the focus as from the directrix.

iii. ✓ **True**

iv. ✗ **False**: $b^2 = a^2(e^2 - 1)$; $1 - e^2$ is the ellipse's.

v. ✗ **False**: It is **inside**, and no tangent can be drawn from it.

vi. ✓ **True**

Each carries 0.5 mark.

3 Multiple choice

i. **b.** $(3, 0)$ — K · 1

$y^2 = 4ax$ with $4a = 12$, $a = 3$: focus $(a, 0) = (3, 0)$. $(-3, 0)$ is on the directrix $x = -3$.

ii. **b.** $\frac{4}{5}$ — U · 1

$b^2 = a^2(1 - e^2)$: $9 = 25(1 - e^2)$, $e^2 = \frac{16}{25}$, $e = \frac{4}{5}$. $\frac{3}{5}$ is $\frac{b}{a}$; $\frac{5}{4}$ is more than 1, a hyperbola's.

iii. **a.** 1 — U · 1

$y = mx + c$ touches $y^2 = 4ax$ when $c = \frac{a}{m}$; here $a = 3$, so $3 = \frac{3}{m}$, $m = 1$.

iv. **a.** 3 — A · 1

$\sqrt{S_1} = \sqrt{16 + 25 - 8 - 20 - 4} = \sqrt{9} = 3$. 9 forgets the square root; $\sqrt{41}$ is the distance from the origin, not from the center.

v. **c.** 90° — HA · 1

$b^2 = a^2(e^2 - 1) = a^2$, so $b = a$ and the asymptotes $y = \pm\frac{b}{a}x = \pm x$ have slopes multiplying to -1: at right angles.

4 Short answers

a. MARKING SCHEME — U · 2

✓ P is the point of the tangent nearest the center (the rest is outside).

✓ The shortest distance to a line is along the perpendicular.

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b. MARKING SCHEME U · 2

✓ The larger denominator is under y^2: the major axis is the Y-axis.

✓ Foci $(0, \pm 4)$.

c. MARKING SCHEME A · 2

✓ Tangent $3x - 4y = 25$.

✓ Normal $4x + 3y = 0$.

d. MARKING SCHEME A · 3

✓ $(y + 2)^2 = 12(x - 1)$.

✓ Vertex $(1, -2)$, $a = 3$.

✓ Focus $(4, -2)$, directrix $x = -2$.

5 Long answers

a. ANSWER A · 5

$a = 3$, $ae = 5$, so $e = \frac{5}{3}$ and $b^2 = (ae)^2 - a^2 = 25 - 9 = 16$: $\boldsymbol{\frac{x^2}{9} - \frac{y^2}{16} = 1}$.

Eccentricity $\frac{5}{3}$. Directrices $x = \pm\frac{a}{e} = \pm\frac{9}{5}$. Latus rectum $\frac{2b^2}{a} = \frac{32}{3}$. Asymptotes $y = \pm\frac{b}{a}x = \pm\frac{4}{3}x$.

MARKING SCHEME

✓ $b^2 = 16$ and the equation.

✓ $e = \frac{5}{3}$.

✓ Directrices $x = \pm\frac{9}{5}$.

✓ Latus rectum $\frac{32}{3}$.

✓ Asymptotes $y = \pm\frac{4}{3}x$.

b. ANSWER A · 8

i. The nearest and farthest points are the ends of the major axis: $2a = 0.59 + 35.1 = 35.69$, $a \approx 17.85$ AU.

ii. $ae = a - 0.59 \approx 17.26$ AU; $e = \frac{17.26}{17.85} \approx 0.967$.

iii. $b^2 = a^2 - (ae)^2 \approx 318.4 - 297.7 = 20.7$, $b \approx 4.55$ AU.

iv. $\frac{x^2}{318.4} + \frac{y^2}{20.7} = 1$ (in AU), the Sun at $(17.26, 0)$ or $(-17.26, 0)$.

v. The Earth's orbit is nearly a circle; the comet's e is close to 1: a long thin ellipse, about four times as long as it is wide, whose near end swings close round the Sun.

MARKING SCHEME

✓ i. $2a =$ the sum of the nearest and farthest distances.

✓ i. $a \approx 17.85$ AU.

✓ ii. $ae = a - 0.59 \approx 17.26$ AU.

✓ ii. $e \approx 0.967$.

✓ iii. $b^2 = a^2 - (ae)^2$.

✓ iii. $b \approx 4.55$ AU.

✓ iv. The equation.

✓ v. e near 1: a long, thin ellipse.

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6 Think

ANSWER HA · 3

From the line, $x = -\dfrac{my + n}{l}$. Put into the parabola: $y^2 = -\dfrac{4a(my + n)}{l}$, that is $ly^2 + 4amy + 4an = 0$.

The line touches when this has equal roots: $(4am)^2 - 4 \cdot l \cdot 4an = 0$, $16a^2m^2 = 16aln$.

So $am^2 = ln$ (dividing by $16a \neq 0$). ✓

MARKING SCHEME

- ✓ The quadratic $ly^2 + 4amy + 4an = 0$.
- ✓ Equal roots: the discriminant 0.
- ✓ $ln = am^2$.

7 Activity: An ellipse with a string

(not marked)

WHAT STUDENTS SHOULD FIND

$2a = 10$ cm (the sum of the distances), $ae = 4$ cm, so $b = \sqrt{25 - 16} = 3$: the minor axis is 6 cm and $e = 0.8$. Moving the pins closer makes e smaller and the curve rounder; with the pins together it is a circle.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	5	9	19	4	**37**

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