Unit 7 · Solution of Triangle

HARDER PRACTICE QUESTIONS

Full marks 27

Write your answers on separate paper. Show your working. Use a calculator where needed.

1 When the sine law misleads [5]

In $\triangle ABC$, $b = 3$, $c = 8$ and $A = 40°$.

a. Find a.

b. Nischal finds C by the sine law: $\sin C = \dfrac{c \sin A}{a}$, and gets $C \approx 58.7°$. Show that he must be wrong, without working out C another way.

c. Solve the triangle correctly, and say which angle it is safe to find first by the sine law, and why.

2 Two trees across a river [6]

Dipesh and Anjila stand at A and B on one bank of a river, 100 m apart. On the far bank are two trees, P and Q, with P nearer to A's end. Their angles, measured from the line AB:

	to P	**to Q**
at A	$\angle PAB = 70°$	$\angle QAB = 35°$
at B	$\angle PBA = 50°$	$\angle QBA = 105°$

a. Find AP from $\triangle PAB$ and AQ from $\triangle QAB$.

b. Find the distance PQ between the trees, which nobody can reach.

c. Explain why the triangle APQ could be solved only after the other two.

3 A slip in the chain [5]

A survey measures a base $AB = 2$ km. In $\triangle ABC$, $\angle A = 58°$ and $\angle B = 72°$. Then in $\triangle BCD$, built on BC, $\angle CBD = 64°$ and $\angle BCD = 66°$. (These give $BC \approx 2.214$ km and $CD \approx 2.598$ km.)

a. The $72°$ was misread; it was really $73°$. Find BC and CD again.

b. Suppose instead the $66°$ in the second triangle was really $67°$ (and the $72°$ was right). Find CD again.

c. Sudarshan says: "An error in one angle only spoils the triangle it was measured in." Use a and b to explain why he is wrong, and why a survey checks its angles most carefully near the base.

4 Consecutive sides [5]

The sides of a triangle are three consecutive whole numbers $n - 1$, n, $n + 1$, and its largest angle is twice its smallest.

a. Let the smallest angle be A. Use the sine law to find $\cos A$ in terms of n.

b. Find $\cos A$ in terms of n another way.

c. Hence find n, and check your triangle.

5 The biggest pen [6]

Sampada has two straight lengths of fence, 30 m and 40 m, joined at a post. She closes the pen with a third straight side, and can choose the angle θ at the post.

a. Write the pen's area in terms of θ, and find the largest area and the angle that gives it. How long is the third side then?

b. Her third length of fence is only 45 m. Find the largest angle θ she can now use, and the largest area.

c. Her brother says: “Put the 40 m and 45 m lengths at the post instead, at a right angle: $\frac{1}{2} \times 40 \times 45 = 900\ \text{m}^2$.” Find his mistake, and show that no choice of which two of the three lengths meet at the post gives a bigger pen than b.