Unit 7 · Solution of Triangle

ANSWER KEY · FOR TEACHERS

Full marks 37

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 4

1. solving
2. shape
3. cosine
4. sum
5. sine
6. acute
7. longest
8. map

Each carries 0.5 mark.

2 True or false

U · 3

i. ✗ **False**: They fix only its shape: triangles of every size have the same angles.
ii. ✓ **True**
iii. ✓ **True**
iv. ✗ **False**: Start with the angle sum for the third angle, then use the sine law.
v. ✗ **False**: It gives angles from $-90°$ to $90°$; $\cos^{-1}$ gives $0°$ to $180°$.
vi. ✓ **True**

Each carries 0.5 mark.

3 Multiple choice

i. **a.** sine law — K · 1

Each other side faces a known angle (the third from the angle sum), and the known side faces one too: $b = \frac{a \sin B}{\sin A}$.

ii. **b.** $\sqrt{39}$ — U · 1

$c^2 = 25 + 49 - 70 \times \frac{1}{2} = 39$. $\sqrt{74}$ forgets the $-2ab\cos C$ term; $\sqrt{109}$ adds it.

iii. **a.** $2\sqrt{3}$ — U · 1

$$b = \frac{a \sin B}{\sin A} = \frac{2\sqrt{2} \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}} = 2\sqrt{3}.$$ $\sqrt{6}$ multiplies by $\sin A$ instead of dividing.

iv. **c.** 95.4 m — A · 1

$\sqrt{2500 + 3600 - 6000\cos 120°} = \sqrt{9100} \approx 95.4$ m. 55.7 m takes $\cos 120° = +\frac{1}{2}$; 78.1 m is for a right angle.

v. **a.** none — HA · 1

$\sin B = \frac{8 \sin 40°}{5} \approx 1.03$, and no sine is more than 1. The side $a = 5$ is too short to reach the other arm of the angle: the nearest it could come is $8 \sin 40° \approx 5.14$.

4 Short answers

a. — U · 2

MARKING SCHEME

✓ The sine law gives θ or $180° - \theta$; $\sin^{-1}$ gives only the acute one.
✓ Only the angle opposite the longest side can be obtuse.

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b. MARKING SCHEME U · 2

✓ C faces the longest side and may be obtuse; she took the acute value.

✓ $C \approx 104.5°$.

c. MARKING SCHEME A · 2

✓ $A = 75°$.

✓ $b \approx 7.99$, $c \approx 11.26$.

d. MARKING SCHEME A · 3

✓ $a = 2\sqrt{7}$.

✓ $B \approx 40.9°$ (the angle opposite the shorter side first).

✓ $C \approx 79.1°$.

5 Long answers

a. ANSWER A · 5

$AC^2 = 6^2 + 4^2 - 2 \times 6 \times 4 \cos 110° = 52 + 16.42 = 68.42$, so $AC \approx 8.27$ km.

$\angle BAC$ faces $BC = 4$, not the longest side, so it is acute: $\sin \angle BAC = \dfrac{4 \sin 110°}{8.27} \approx 0.454$,

$\angle BAC \approx 27.0°$.

(Check: $\angle ACB \approx 180° - 110° - 27.0° = 43.0°$, and $\dfrac{6}{\sin 43.0°} \approx \dfrac{8.27}{\sin 110°} \approx 8.80$. ✓)

MARKING SCHEME

✓ The cosine law with $\cos 110°$ negative.

✓ $AC \approx 8.27$ km.

✓ The angle opposite the shorter side chosen for the sine law.

✓ $\sin \angle BAC \approx 0.454$.

✓ $\angle BAC \approx 27.0°$.

b. ANSWER A · 8

i. $\angle ACB = 180° - 58° - 72° = 50°$. $BC = \dfrac{2 \sin 58°}{\sin 50°} \approx 2.214$ km; $AC = \dfrac{2 \sin 72°}{\sin 50°} \approx 2.483$ km.

ii. $\angle BDC = 180° - 64° - 66° = 50°$. $CD = \dfrac{BC \sin 64°}{\sin 50°} \approx 2.598$ km; $BD = \dfrac{BC \sin 66°}{\sin 50°} \approx 2.640$ km.

iii. Only the base AB was measured; every other length came from the angles, the second triangle starting from BC, worked out in the first.

MARKING SCHEME

✓ i. $50°$.

✓ i. $BC \approx 2.214$ km.

✓ i. $AC \approx 2.483$ km.

✓ ii. $50°$.

✓ ii. BC used as the known side.

✓ ii. $CD \approx 2.598$ km.

✓ ii. $BD \approx 2.640$ km.

✓ iii. Only AB measured.

6 Think

ANSWER HA · 3

$c^2 = a^2 + b^2 - 2ab \cos 60° = 4b^2 + b^2 - 2b^2 = 3b^2$, so $c = \sqrt{3}b$.

Then $b^2 + c^2 = b^2 + 3b^2 = 4b^2 = a^2$: by the converse of Pythagoras, $A = 90°$.

So $A = 90°$, $C = 60°$ and $B = 30°$ (check: $\sin B = \dfrac{b}{a} = \dfrac{1}{2}$).

MARKING SCHEME

✓ $c^2 = 3b^2$.

✓ $b^2 + c^2 = a^2$, so $A = 90°$.

✓ $B = 30°$.

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7 Activity: Solve it, then measure it

(not marked)

WHAT STUDENTS SHOULD FIND

$\angle C = 70°$; $AC = \dfrac{8 \sin 60°}{\sin 70°} \approx 7.37$ cm and $BC = \dfrac{8 \sin 50°}{\sin 70°} \approx 6.52$ cm. A careful drawing agrees to within a millimeter or two. One side and two angles: the angle sum, then the sine law.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	5	9	19	4	**37**

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