Unit 6 · Properties of a Triangle

WORKSHEET

Full marks 37

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 37

1 Key words [4]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

cosine	**half-angle**	**Heron's**	**induction**	**opposite**	**pivot**	**projection**	**semi-perimeter**
sine	**tangent**						

In any triangle, each side is in proportion to the sine of the angle _____ it: this is the _____ law. From three sides, the _____ law gives every angle. Given two sides and the angle between them, the _____ law gives the difference of the other two angles.

$a = b\cos C + c\cos B$ is a _____ law: each side is the sum of the shadows of the other two on it.

Half the perimeter, s, is the _____; it appears in the _____ laws and in _____ formula for the area.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. In every triangle, the longest side faces the largest angle. ☐

ii. If $a^2 > b^2 + c^2$, the angle A is acute. ☐

iii. $a = b\cos C + c\cos B$ in every triangle. ☐

iv. The sine law can find the angles of a triangle from its three sides alone. ☐

v. The area of a triangle is $\frac{1}{2}ab\sin C$. ☐

vi. $\cos\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$. ☐

3 Multiple choice [5]

Choose the correct option for each of the following questions.

i. In $\triangle ABC$, $\frac{a}{\sin A}$ is equal to: [1]

a. $\frac{\sin B}{b}$ **b.** $\frac{c}{\sin C}$

c. $\frac{a}{\cos A}$ **d.** $b\sin B$

ii. In $\triangle ABC$, $b = 3$, $c = 5$ and $A = 60°$. Then a is: [1]

a. 7 **b.** $\sqrt{19}$

c. $\sqrt{34}$ **d.** 4

iii. The largest angle of the triangle with sides 7, 8 and 13 is: [1]

a. 60° **b.** 90°

c. 120° **d.** 150°

iv. The area of the triangle with sides 7, 8 and 9 is: [1]

a. 24 **b.** $12\sqrt{5}$

c. $6\sqrt{5}$ **d.** 36

v. In $\triangle ABC$, $a \cos B = b \cos A$. The triangle is: [1]

a. isosceles **b.** right-angled

c. equilateral **d.** isosceles or right-angled

4 Short answers [9]

Write short answers to the following questions.

a. Explain why the cosine law $a^2 = b^2 + c^2 - 2bc \cos A$ can be called "Pythagoras's theorem with a correction". What does the sign of the correction say about A? [2]

b. To find the angle A of a triangle with sides $a = 6$, $b = 4$, $c = 5$, Binod wrote $\cos A = \frac{a^2 + b^2 - c^2}{2ab}$. [2]

Find his mistake and the right value of $\cos A$.

c. Find the area of the triangle with sides 5 cm, 6 cm and 7 cm. [2]

d. Prove that $a(b \cos C - c \cos B) = b^2 - c^2$. [3]

5 Long answers [13]

Write long answers to the following questions. Show your working.

a. Prove that in any triangle $(b - c)\cos\frac{A}{2} = a\sin\frac{B - C}{2}$. [5]

b. Two students, Sabina and Ganesh, stand at A and B on one bank of a river, 500 m apart. A tree C stands on the far bank. Sabina measures $\angle CAB = 70°$ and Ganesh $\angle CBA = 60°$. [8]

i. Find the angle at the tree, $\angle ACB$.
ii. Find the distance AC by the sine law.
iii. Find the distance BC.
iv. Find the width of the river: the distance from C to the line AB.
v. Check your answers with the projection law on AB.



Answer:

6 Think [3]

Prove that in any triangle $b \cos B + c \cos C = a \cos (B - C)$.

7 Activity: A triangle from its sides (not marked)

With a ruler and compasses, draw a triangle with sides 4 cm, 5 cm and 6 cm. Measure its three angles with a protractor, then work them out with the cosine law.

Angle	Facing the side	Measured	By the cosine law
smallest	4 cm		
middle	5 cm		
largest	6 cm		
total			

What do you notice about the largest angle and the smallest?