Unit 5 · Matrix Based System of Linear Equations

HARDER PRACTICE QUESTIONS

Full marks 28

Write your answers on separate paper. Show your working.

1 When the determinant is zero [6]

Consider the system $kx + y + z = 1, x + ky + z = 1, x + y + kz = 1$.

a. Show that its determinant is $D = (k - 1)^2(k + 2)$.

b. Solve the system when $D \neq 0$.

c. Anjila says: "When $D = 0$, there is no solution." Test her claim at each value of k that makes $D = 0$.

2 A curve through measured points [6]

Bikram throws a ball straight up from his hand at $t = 0$, and a friend films it. From the video he reads its height h (in meters above his hand) at four times t (in seconds): 25 m at 1 s, 40 m at 2 s, 45 m at 3 s and 42 m at 4 s. (The readings are made up.) The ball's height is $h = at^2 + bt + c$.

a. Show that no such curve passes through all four readings.

b. Find the curve through the first three readings, and the curve through the last three.

c. Suppose only one reading is wrong. Use the fact that the ball left his hand at $t = 0$ to decide which one, and give the ball's greatest height.

3 Two industries that buy from each other [6]

In a simple model of an economy there are two industries, farming and manufacturing. To produce goods worth Rs 1, farming uses Rs 0.40 of farm goods and Rs 0.40 of manufactured goods; manufacturing uses Rs 0.20 of farm goods and Rs 0.20 of manufactured goods. Besides what they use, they must deliver a final demand (to households and for export) of Rs 50 crore of farm goods and Rs 100 crore of manufactured goods. (The numbers are made up.)

a. If farming produces x_1 and manufacturing x_2 (in Rs crore), write the two equations, and find x_1 and x_2 by the inverse of the coefficient matrix.

b. The planners can add Rs 10 crore to the final demand for farm goods or for manufactured goods. Smriti says that either way the two industries together must produce Rs 10 crore more. Without solving the system again, find what each choice really needs, and say which needs more.

c. Farming can produce at most Rs 200 crore. With farm final demand kept at Rs 50 crore, what is the largest final demand for manufactured goods that can be met?

4 One matrix, two right sides [6]

$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 1 & 3 & 4 \end{pmatrix}$.

a. Try to find A^{-1} by row operations on $[A \mid I]$. Show where and why the method stops.

b. Solve $AX = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$.

c. Show that $AX = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}$ has no solution.

d. What must be true of b_1, b_2, b_3 for $AX = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ to have a solution?

5 Two operations at once [4]

Kiran solves $x + y = 3, x - y = 1$. He does $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_1$ at the same time, each from the old rows, and gets the rows $(0\ 2 \mid 2)$ and $(0\ -2 \mid -2)$. He concludes that $y = 1$ and x can be anything.

a. Find the true solution.

b. Explain exactly what went wrong. Why is each of his two operations allowed on its own, but not the two done together this way?

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