Unit 4 · Sequence and Series

WORKSHEET

Full marks 37

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 37

1 Key words [4]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

base	**binomial**	**cubes**	**hypothesis**	**induction**	**inductive**	**matrix**	**square**	**sum**
telescope								

A sum in which every term but the first and the last cancels is said to _____. The sign Σ is read "the _____ of". A statement about every natural number can be proved by mathematical _____. Its first step, showing the statement for $n = 1$, is the _____ step. In the second step, the statement for k is assumed: this is the induction _____, and the step that proves it for $k + 1$ is the _____ step. The sum of the first n _____ is the square of the sum of the first n natural numbers, and the sum of the first n odd numbers is a perfect _____.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. $1 + 3 + 5 + \cdots + (2n - 1) = n^2$. ☐

ii. $1^2 + 2^2 + \cdots + n^2 = (1 + 2 + \cdots + n)^2$. ☐

iii. $\Sigma_{k=1}^{n} 5 = 5$. ☐

iv. A formula checked for $n = 1, 2, \ldots, 100$ is not yet proved for every n. ☐

v. In a proof by induction, the inductive step alone is enough. ☐

vi. $1^3 + 2^3 + \cdots + n^3$ is a perfect square for every n. ☐

3 Multiple choice [5]

Choose the correct option for each of the following questions.

i. $1 + 2 + 3 + \cdots + n$ equals: [1]

a. n^2

b. $\frac{n(n+1)}{2}$

c. $n(n + 1)$

d. $\frac{n(n+1)(2n+1)}{6}$

ii. $\Sigma_{k=1}^{20} (2k + 1)$ equals: [1]

a. 440

b. 421

c. 420

d. 441

iii. The nth term of $2 \cdot 4 + 4 \cdot 7 + 6 \cdot 10 + \cdots$ is: [1]

a. $2n(3n + 4)$ **b.** $2n(3n + 1)$

c. $n(3n + 1)$ **d.** $2n(n + 2)$

iv. If $1^2 + 2^2 + \cdots + n^2 = 650$, then n is: [1]

a. 10 **b.** 12

c. 13 **d.** 25

v. $1 \cdot n + 2(n - 1) + 3(n - 2) + \cdots + n \cdot 1$ equals: [1]

a. $\frac{n(n+1)(n+2)}{6}$ **b.** $\frac{n(n+1)}{2}$

c. $\frac{n^2(n+1)}{2}$ **d.** $\frac{n(n+1)(2n+1)}{6}$

4 Short answers [9]

Write short answers to the following questions.

a. A proof by induction is often compared with a line of dominoes. Explain what the two steps of the proof are in this picture, and why both are needed. [2]

b. Puja wrote $\Sigma_{k=1}^{10} (k^2 + 3) = 385 + 3 = 388$. Find her mistake and the right answer. [2]

c. Find $21^2 + 22^2 + \cdots + 30^2$. [2]

d. Find the sum of n terms of $1 \cdot 4 + 2 \cdot 7 + 3 \cdot 10 + \cdots$, and check your answer for $n = 2$. [3]

5 Long answers [13]

Write long answers to the following questions. Show your working.

a. Prove by mathematical induction that $2 + 6 + 18 + \cdots + 2 \cdot 3^{n-1} = 3^n - 1$ for every natural number n. [5]

b. Consider the series $1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \cdots$ [8]

i. Write its nth term.

ii. Find the sum of its first n terms.

iii. Check your formula for $n = 2$.

iv. Find the sum of the first 10 terms.

v. How many terms must be added for the sum to be more than 1000?

Answer:

6 Think [3]

Take the false formula $1 + 2 + \cdots + n = \dfrac{n^2 + n + 2}{2}$. Show that its inductive step works: if it were true for k, it would be true for $k + 1$. Then explain why it is still false.

7 Activity: Cubes and squares (not marked)

Fill in the table for $n = 1$ to 6. Then, on squared paper, draw a square of side $1 + 2 + 3 = 6$ and cut it into L-shaped pieces of 1, 8 and 27 small squares.

n	$1 + 2 + \cdots + n$	its square	$1^3 + 2^3 + \cdots + n^3$
1			
2			
3			
4			
5			
6			

What do you notice? Does your table prove it for every n?