Unit 3 · Complex Numbers

WORKSHEET

Full marks 37

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 37

1 Key words [4]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

argument	**asymptote**	**conjugate**	**De Moivre's**	**determinant**	**Euler's**	**modulus**	**polar**
principal	**unity**						

For $z = a + bi$, the distance $r = \sqrt{a^2 + b^2}$ from the origin is the _____ of z, and the angle θ its line makes with the positive real axis is its _____; the one in $-\pi < \theta \leq \pi$ is the _____ argument. Writing $z = r(\cos\theta + i\sin\theta)$ is the _____ form. The rule $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ is _____ theorem. The three numbers whose cube is 1 are the cube roots of _____. The number $a - bi$ is the _____ of $a + bi$. The rule $e^{i\theta} = \cos\theta + i\sin\theta$ is _____ formula.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. $|z_1 z_2| = |z_1|\,|z_2|$. ☐

ii. A non-zero complex number has exactly two cube roots. ☐

iii. $1 + \omega + \omega^2 = 0$, where ω is a complex cube root of unity. ☐

iv. $(\cos\theta + i\sin\theta)^{-1} = \cos\theta + i\sin\theta$. ☐

v. $e^{i\pi} = -1$. ☐

vi. To multiply two complex numbers in polar form, multiply their arguments. ☐

3 Multiple choice [5]

Choose the correct option for each of the following questions.

i. The modulus of $3 - 4i$ is: [1]

a. 5 **b.** 7
c. 25 **d.** 1

ii. z_1 has argument 100° and z_2 has argument 130°. The principal argument of $z_1 z_2$ is: [1]

a. 230° **b.** -130°
c. 30° **d.** 130°

iii. $(1 + i)^8$ is equal to: [1]

a. 16 **b.** -16
c. $16i$ **d.** 8

iv. If ω is a complex cube root of unity, $\omega^{100} + \omega^{200} + 1$ is: [1]

a. 0 **b.** 3

c. 1 **d.** -1

v. How many of the twelve 12th roots of unity are also 8th roots of unity? [1]

a. 2 **b.** 4

c. 8 **d.** 12

4 Short answers [9]

Write short answers to the following questions.

a. Sanjeev says the principal argument of $-1 - i$ is $\tan^{-1}\frac{-1}{-1} = \tan^{-1} 1 = \frac{\pi}{4}$. What is wrong, and what is the right value? [2]

b. Rohan wrote $(\cos\theta + i\sin\theta)^2 = \cos^2\theta + i\sin^2\theta$. Correct him, and say what the correct result tells you about $\cos 2\theta$ and $\sin 2\theta$. [2]

c. Find the modulus and the principal argument of $(\sqrt{3} + i)(1 + i)$ without multiplying out. [2]

d. If ω is a complex cube root of unity, show that $(1 - \omega + \omega^2)(1 + \omega - \omega^2) = 4$. [3]

5 Long answers [13]

Write long answers to the following questions. Show your working.

a. Find the three cube roots of -27. Show that they are the corners of an equilateral triangle, and find their sum. **[5]**

b. Let $z = 1 - \sqrt{3}\,i$. **[8]**

i. Express z in polar form and in the form $re^{i\theta}$.

ii. Use De Moivre's theorem to find z^{10} in the form $a + bi$.

iii. Show that z^3 is a real number, and find it.

iv. From Euler's formula, show that $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$.

Answer:

6 Think [3]

Use the cube roots of unity to show that $a^3 + b^3 = (a + b)(a + b\omega)(a + b\omega^2)$.

7 Activity: Multiplying by i is a quarter turn (not marked)

On graph paper, with 1 cm for 1 unit, plot $z = 3 + 2i$, then iz, i^2z and i^3z. Measure each point's distance from the origin and the angle its line makes with the positive real axis.

Number	In the form $a + bi$	Distance from O (cm)	Angle
z	$3 + 2i$		
iz			
i^2z			
i^3z			

What happens to the distance and the angle each time you multiply by i? What would i^4z be?