Unit 2 · Binomial Theorem

WORKSHEET

Full marks 37

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 37

1 Key words [4]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

approximation	**binomial**	**Euler's**	**factorial**	**general**	**independent**	**index**	**middle**
Pascal's	**permutation**						

A sum of two terms, such as $a + b$, is a _____. The term $t_{r+1} = C(n, r)\,a^{n-r}b^r$ is the _____ term of the expansion of $(a + b)^n$. When n is even, the expansion has one _____ term. A term with no x in it is called the term _____ of x. Each number in _____ triangle is the sum of the two above it. For a power that is not a positive whole number, the binomial theorem for any _____ gives a series that never ends. For small x, $(1 + x)^n \approx 1 + nx$ is an _____. The sum $1 + \frac{1}{1!} + \frac{1}{2!} + \cdots$ is _____ number e.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. The expansion of $(a + b)^n$, n a positive integer, has $n + 1$ terms. ☐

ii. The series for $(1 + x)^{-2}$ equals $(1 + x)^{-2}$ for every x. ☐

iii. The 5th term of $(1 + x)^n$ is $C(n, 5)x^5$. ☐

iv. The sum of the coefficients in the expansion of $(3x - 2)^7$ is 1. ☐

v. The series $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ equals e^x only when $|x| < 1$. ☐

vi. $(1.02)^5$ is 1.10 to two decimal places. ☐

3 Multiple choice [5]

Choose the correct option for each of the following questions.

i. The number of terms in the expansion of $(2x - 3y)^{10}$ is: [1]

a. 10 **b.** 11

c. 12 **d.** 20

ii. The term independent of x in the expansion of $\left(x + \frac{1}{x}\right)^6$ is: [1]

a. 20 **b.** 15

c. 6 **d.** 1

iii. The coefficient of x^3 in the expansion of $(1+x)^{-2}$ is: [1]

a. 4 **b.** -4

c. -6 **d.** 10

iv. By the binomial approximation, $\frac{1}{\sqrt{1.01}}$ is about: [1]

a. 0.995 **b.** 0.990

c. 1.005 **d.** 0.950

v. Without a calculator, which is true? [1]

a. $(1.01)^{100} > 2$ **b.** $(1.01)^{100} = 2$

c. $(1.01)^{100} < 2$ **d.** It can't be decided without a calculator.

4 Short answers [9]

Write short answers to the following questions.

a. Explain why the expansion of $(1+x)^5$ stops after six terms, while the expansion of $(1+x)^{1/2}$ never stops. [2]

b. Anisha wrote: "$(4+x)^{1/2} = 4\left(1+\frac{x}{4}\right)^{1/2} \approx 4+\frac{x}{2}$, valid for $|x|<1$." Find her two mistakes and correct them. [2]

c. Find the coefficient of x^7 in the expansion of $\left(x^2+\frac{2}{x}\right)^8$. [2]

d. The coefficients of the 2nd, 3rd and 4th terms in the expansion of $(1+x)^n$ are in arithmetic progression. Find n. [3]

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https://science-playground.com/learn/grade-12-math/binomial-theorem

5 Long answers [13]

Write long answers to the following questions. Show your working.

a. [5]

i. Expand $(1 - x)^{-1/2}$ as far as the term in x^3, and say for which x the expansion is valid.

ii. Using $x = 0.02$, find $\frac{1}{\sqrt{0.98}}$ correct to 4 decimal places.

b. [8]

In the expansion of $\left(2x^2 - \frac{1}{3x}\right)^9$:

i. write the general term;

ii. find the term independent of x;

iii. find the coefficient of x^3;

iv. find the two middle terms.

v. Write the series for e^x, and from it find $e^{-0.1}$ correct to 4 decimal places.

Answer:

6 Think [3]

Without a calculator, show that $(1.1)^{10} > 2.5$.

7 Activity: How far can 1 + nx be trusted? (not marked)

Use a calculator for $(1.01)^n$, and the first two and the first three terms of the binomial theorem for the approximations.

n	$(1.01)^n$	$1 + 0.01n$	$1 + 0.01n + \frac{n(n-1)}{2}(0.0001)$
5			
10			
50			
100			

For which n does $1 + 0.01n$ stop being good to 2 decimal places? Why does it get worse as n grows?