Unit 2 · Binomial Theorem

HARDER PRACTICE QUESTIONS

Full marks 28

Write your answers on separate paper. Show your working. Here log means $\log_e$, and $\log 2 = 0.6931$.

1 A binomial with an extra factor [4]

In the expansion of $(1 + x)\left(x - \frac{2}{x}\right)^8$:

a. find the term independent of x;

b. find the coefficient of x.

c. Prabin says that in a both factors of $(1 + x)$ give a part of the answer. Is he right? Explain.

2 The greatest coefficient [4]

a. Which coefficient is the greatest in the expansion of $(1 + x)^{10}$?

b. Bishnu says the greatest coefficient in the expansion of $(1 + 2x)^{10}$ is also the middle one, $C(10, 5) \times 2^5 = 8064$. Find the greatest coefficient, and say where his reasoning fails.

3 Two series multiplied [5]

a. Expand $\frac{\sqrt{1+x}}{1-x}$ in powers of x as far as the term in x^2. For which x is the expansion valid?

b. Use it to find $\frac{\sqrt{1.01}}{0.99}$ correct to 5 decimal places, and show that the terms up to x^2 are enough for that.

c. Why would the same three terms give a poor value of $\frac{\sqrt{1.5}}{0.5}$?

4 Errors that pull opposite ways [5]

A water tank is a cylinder, $V = \pi r^2 h$. Its radius is measured 2% too large and its height 3% too small.

a. Use the binomial approximation to estimate the percentage error in the volume worked out from these measurements.

b. Find the error exactly, and explain why the estimate in a is a little off.

c. Kopila says: “The radius is 2% out and the height 3% out, so the volume is at least 3% out.” Explain why she is wrong.

5 Why 72? [6]

Money at R% interest a year, added once a year, doubles in $n = \frac{\log 2}{\log(1 + r)}$ years, where $r = \frac{R}{100}$; the logarithmic series and the binomial approximation turn this into $n \approx \frac{69.31}{R} + 0.35$.

a. Another bank pays the same R% a year, but adds $\frac{R}{2}$% every half year. Show that money there doubles in about $\frac{69.31}{R} + 0.17$ years.

b. At what rate R does the once-a-year formula give exactly the same answer as the rule of 72, $n \approx \frac{72}{R}$?

c. Gyanu’s savings earn 18% a year, added once a year. The exact doubling time is 4.19 years. Which is closer, the rule of 72 or the once-a-year formula? A cooperative pays 2%, where the exact time is 35.0 years. Which is closer there? Explain what you see.

6 The squares of the coefficients [4]

a. By finding the coefficient of x^n on both sides of $(1+x)^n(1+x)^n = (1+x)^{2n}$, show that

$$C(n,0)^2 + C(n,1)^2 + C(n,2)^2 + \cdots + C(n,n)^2 = C(2n,n)$$

b. Check it for $n = 4$.

c. A class has 4 boys and 4 girls, and 4 of the 8 are to be chosen. Explain the identity for $n = 4$ by counting the choices by the number of boys chosen.