Grade 12 Mathematics, Explained

# Unit 1 · Permutation and Combination

**HARDER PRACTICE QUESTIONS**

Full marks 29

Write your answers on separate paper. Show your working. Names, places and numbers are made up.

## 1 "At least one": a count that came out too big [5]

A committee of 4 is chosen from 6 boys and 4 girls, with at least one girl on it. Shristi reasons: "First choose one girl, $C(4, 1) = 4$ ways, so the condition is met. The other 3 members can be anyone from the 9 left, $C(9, 3) = 84$ ways. So there are $4 \times 84 = 336$ committees."

a. Find the correct number of committees.

b. Shristi's answer is too big. How many times has she counted a committee with exactly 2 girls? With exactly $k$ girls?

c. Show that her 336 is exactly what you get when every committee is counted as many times as you said in b.

## 2 A round table with two conditions [5]

Five delegates from Nepal and five from India sit at a round table for a meeting, Nepalis and Indians in alternate seats.

a. In how many ways can they sit?

b. Two of them, Pemba from Nepal and Arjun from India, must sit side by side. In how many ways can the ten sit now?

c. Without using your answer to b, explain why exactly $\frac{2}{5}$ of the seatings in a have Pemba and Arjun side by side.

d. If the ten sat in a row of ten chairs instead, still alternating, how many seatings would there be?

## 3 Routes through the old town [5]

The lanes of part of an old town are drawn as a grid of squares. Kamala walks from the crossing $A$ at the south-west corner to the crossing $B$, which is 6 blocks east and 4 blocks north of $A$, always walking east or north. The crossing $P$ is 2 blocks east and 2 blocks north of $A$, and $Q$ is the next crossing east of $P$.

a. How many shortest routes are there from $A$ to $B$?

b. How many of them pass through $P$?

c. One day the crossing $P$ is closed for a festival. How many routes are left?

d. Another day only the lane from $P$ to $Q$ is closed; Kamala may still walk through $P$. Kiran says the answer is the same as in c. Is he right? Find the number of routes.

## 4 Groups with and without names [6]

Twelve students are to be split into three groups of 4 for a project.

a. In how many ways can it be done if the groups get different topics (water, air and soil)?

b. In how many ways if the groups get no topics, so that only who is with whom matters? Explain why your answer is not the same as in a.

c. Tashi and Gaurav are two of the students. In how many of the splits of b are they in the same group?

d. Check your answer to c by thinking about where Gaurav can be once Tashi's place is fixed.

## 5 Adding up all the numbers [4]

a. Without listing them, find the sum of all the four-digit numbers that can be made from the digits 1, 2, 3, 4, each used once.

b. Find the sum of all the four-digit numbers that can be made from 0, 1, 2, 3, each used once. (A number can't start with 0.)

## 6 Fruits that are alike [4]

A basket has 3 apples, 4 oranges and 2 mangoes. Fruits of one kind are alike.

a. Nima says the number of ways to take some fruit (at least one) is $2^9 - 1 = 511$, since each of the 9 fruits is either taken or not. Why is he wrong?

b. Find the right number.

c. How many selections have at least one fruit of each kind?