Unit 1 · Permutation and Combination

ANSWER KEY · FOR TEACHERS

Full marks 37

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 4

1. permutation
2. combination
3. factorial
4. counting
5. circular
6. necklace
7. Pascal's
8. binomial

Each carries 0.5 mark.

2 True or false

U · 3

i. ✓ **True**

ii. ✗ **False**: In $(6-1)! = 120$ ways: turning everyone one seat round gives the same seating.

iii. ✓ **True**

iv. ✗ **False**: It is $10^3 = 1000$; $P(10, 3) = 720$ counts only the PINs with three different digits.

v. ✗ **False**: The two jobs are different, so order matters: $P(11, 2) = 110$.

vi. ✓ **True**

Each carries 0.5 mark.

3 Multiple choice

i. **a.** $C(n+1, r)$ — K · 1

Pascal's rule: choose r from $n+1$ people, one particular person left out ($C(n, r)$) or in ($C(n, r-1)$).

ii. **b.** $\frac{9!}{2!}$ — U · 1

Nine letters, A twice and the others (K, T, H, M, N, D, U) once: $\frac{9!}{2!} = 1{,}81{,}440$.

iii. **c.** 4320 — U · 1

The girls as one block and the 5 boys are 6 things: $6! = 720$; inside the block $3! = 6$: $720 \times 6 = 4320$. 720 forgets the order inside the block; 40,320 is 8!, no condition at all.

iv. **b.** 40 — A · 1

Any two points: $C(10, 2) = 45$. The $C(4, 2) = 6$ pairs from the collinear points all give the same line, so take 6 away and add that one line back: $45 - 6 + 1 = 40$. 39 forgets to add it back.

v. **c.** 56 — HA · 1

Leave the 7 chairs not chosen in a row: they make 8 gaps (between them and at the two ends), and the 3 chosen chairs go into 3 different gaps: $C(8, 3) = 56$. 120 is $C(10, 3)$, with no condition.

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4 Short answers

a. MARKING SCHEME U · 2

- ✓ Rotating a seating doesn't change it (only the neighbors matter).
- ✓ Each seating gives n row arrangements, so $n!/n = (n-1)!$ (or fix one person, arrange the rest).

b. MARKING SCHEME A · 2

- ✓ $(n-2)(n-3) = 20$.
- ✓ $n = 7$.

c. MARKING SCHEME U · 2

- ✓ His count is "not all three together", which still has two vowels side by side.
- ✓ The gap method: $180 \times 35 \times 3 = 18{,}900$.

d. MARKING SCHEME A · 3

- ✓ $\dfrac{n-r+1}{r} = \dfrac{7}{3}$ (from the ratio of consecutive coefficients).
- ✓ $\dfrac{n-r}{r+1} = \dfrac{3}{2}$.
- ✓ $r = 3$, $n = 9$.

5 Long answers

a. ANSWER A · 5

i. Choose the 2 women and then the 4 men: $C(5, 2) \times C(7, 4) = 10 \times 35 = 350$.

ii. All committees: $C(12, 6) = 924$. With no woman: $C(7, 6) = 7$; with one: $C(5, 1) \times C(7, 5) = 5 \times 21 = 105$. So $924 - 7 - 105 = 812$.

iii. Committees with both: the other 4 from the remaining 10, $C(10, 4) = 210$. So $924 - 210 = 714$.

MARKING SCHEME

- ✓ i. $C(5, 2) \times C(7, 4) = 350$.
- ✓ ii. Total $C(12, 6) = 924$ (or the cases 2, 3, 4, 5 women written).
- ✓ ii. $924 - 7 - 105 = 812$.
- ✓ iii. Both serve: $C(10, 4) = 210$.
- ✓ iii. $924 - 210 = 714$.

b. ANSWER A · 8

The first place can't be 0: fill it first.

i. $5 \times 5 \times 4 \times 3 \times 2 = 600$.

ii. Split by the last digit, since 0 touches both restrictions. Last digit 0: $5 \times 4 \times 3 \times 2 = 120$. Last digit 2 or 4: the first place has 4 choices (not 0, not the last digit), then $4 \times 3 \times 2$: $2 \times 4 \times 4 \times 3 \times 2 = 192$. Even: $120 + 192 = 312$.

iii. Last digit 0: 120. Last digit 5: $4 \times 4 \times 3 \times 2 = 96$. So 216.

iv. The first digit is 4 or 5, and the other four places are filled from the 5 digits left: $2 \times 5 \times 4 \times 3 \times 2 = 240$.

MARKING SCHEME

- ✓ i. $5 \times 5 \times 4 \times 3 \times 2 = 600$.
- ✓ ii. Split into the cases last digit 0 and last digit 2 or 4.
- ✓ ii. Last digit 0: 120.
- ✓ ii. Last digit 2 or 4: 192; even 312.
- ✓ iii. Last digit 0 (120) or 5 (96).
- ✓ iii. 216.
- ✓ iv. First digit 4 or 5 (2 ways), the rest from 5 digits.
- ✓ iv. 240.

6 Think

ANSWER

Choose 2 of the $2n$ students: $C(2n, 2)$ ways. Or split by who is chosen: two boys, $C(n, 2)$; two girls, $C(n, 2)$; a boy and a girl, $n \times n = n^2$. The three cases don't overlap and cover every pair, so

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$C(2n, 2) = 2\,C(n, 2) + n^2$.

For $n = 7$: $C(14, 2) = 91$, and $2 \times 21 + 49 = 91$. ✓

MARKING SCHEME

✓ $C(2n, 2)$ counts all pairs.

✓ The cases two boys, two girls, one of each: $C(n, 2) + C(n, 2) + n^2$, with no overlap.

✓ The check: $91 = 42 + 49$.

7 Activity: Shortest routes on a grid of streets

(not marked)

WHAT STUDENTS SHOULD FIND

Every route is a word of 3 E's and 2 N's: 6 start with E and 4 with N, 10 in all. That is $\frac{5!}{3!\,2!} = 10$, the arrangements of letters not all different, which is also $C(5, 2)$: choosing which 2 of the 5 steps go north.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	5	9	19	4	**37**

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