Unit 18 · Dynamics

STUDY NOTES

WHAT NEB'S GRADE 12 EXAM ASKS

- **CDC's grid (2078):** Mechanics (Units 17–18) **or** Computational Methods (Units 15–16), as the school teaches: 6 of 75 marks; 1 MCQ (an "OR" in the paper) and 1 short question; no long question
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours
- **CDC's model paper:** the short question split 3 + 2; its 3-mark part a bullet stopped in a tree trunk; a part asks a resisting force or distance, the acceleration and tension of joined bodies, or a projectile's time, height, range or angle
- **MCQ (1 mark)**
  - Newton's second law
  - the range formula
  - velocity from a force and time
  - the other angle, same range
  - at the highest point
  - height from a 45° range
  - table and pulley: a
  - a bullet's resistance
  - thrown level from a tower
  - range = greatest height
  - speed from the top point
- **Short and long (5 or 8 marks, in parts)**
  - a car braking; hitting a wall; a seat belt
  - two masses over a pulley
  - a projectile at one height twice
  - when it moves at 30°
  - derive the path; vertex, latus rectum; through a point

Key terms

Momentum $p = mv$: a vector along v.

Impulse: $\int F\,dt$, the change of momentum; Ft for a steady force.

1 newton: gives 1 kg an acceleration of 1 m/s^2; 1 kg wt = 9.8 N.

Light, inextensible string; smooth pulley: one tension all along, one acceleration.

Projectile: a body thrown and left to gravity; **angle of projection** θ above the horizontal.

Trajectory; time of flight; range; greatest height: its path; time in the air; level distance; top height.

Newton's laws

1. **First**: a body stays at rest or in uniform motion in a straight line unless an external (resultant) force acts
2. **Second**: the rate of change of momentum is proportional to the resultant force, in its direction
3. **Third**: to every action an equal and opposite reaction, along the same line

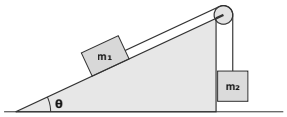
- $F = d(mv)/dt$ (the newton makes the constant 1); constant mass: $F = ma$
- weight mg
- integrate: impulse $\int_{t_1}^{t_2} F\,dt = mv - mu$
- third law → impulses cancel → **momentum conserved**
- force varying with time: integrate $m\,dv/dt = F(t)$
  - 2 kg, $F = 4t$: $v = t^2$; at 3 s, 9 m/s, 9 m
- uniform: $v = u + at$, $s = ut + at^2/2$, $v^2 = u^2 + 2as$, $s = (u + v)t/2$
- bullet 20 g, 400 m/s, stopped in 1/2000 s: $a = 8 \times 10^5$ m/s^2, $F = 16\,000$ N, 10 cm
- stopped in a distance: $v^2 = u^2 + 2as$, then $F = ma$

Bodies joined by a string

- one body at a time: $F = ma$ along its motion; then add
- table and pulley (m_1 on a smooth table, m_2 hanging)
  - $m_2g - T = m_2a$; $T = m_1a$

- $a = m_2 g/(m_1 + m_2)$; $T = m_1 m_2 g/(m_1 + m_2)$
- $T < m_2 g$: it falls slower than g

- two hanging masses (Atwood): $a = (m_1 - m_2)g/(m_1 + m_2)$; force on the pulley $2T$
- smooth slope at θ, pulley at the top: along the slope only $m_1 g \sin\theta$
  - 5 kg at 30°, 3 kg hanging: $29.4 > 24.5$ N $\rightarrow$ $a = 0.6125$ m/s^2, $T \approx 27.56$ N



Block on a smooth slope at θ, over the pulley to a hanging mass.

Projectile

From the accelerations (no air; up positive):

1. $d^2x/dt^2 = 0$, $d^2y/dt^2 = -g$ (only the weight)
2. $dx/dt = u\cos\theta$, $dy/dt = u\sin\theta - gt$
3. $x = ut\cos\theta$, $y = ut\sin\theta - gt^2/2$
4. eliminate t:

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

- $y = bx - cx^2$: a **parabola**, vertical axis, opening down
  - complete the square: vertex at the top $(R/2, H)$; latus rectum $2u^2\cos^2\theta/g$

$T = \dfrac{2u\sin\theta}{g}$	lands at its start's level **Use:** time of flight
$H = \dfrac{u^2\sin^2\theta}{2g}$	at $x = R/2$ **Use:** greatest height
$R = \dfrac{u^2\sin 2\theta}{g}$	$\le u^2/g$, at 45° **Use:** range
$\tan\varphi = \dfrac{u\sin\theta - gt}{u\cos\theta}$	horizontal part never changes **Use:** direction at t

- θ and $90° - \theta$: **the same range** (flat and high)
  - 70 m at 30 m/s: 24.8° (2.57 s, 8.1 m) or 65.2° (5.56 s, 37.8 m)
- at the top: velocity horizontal, $u\cos\theta$; not zero
- **through a point**: put it in the path; $1/\cos^2\theta = 1 + \tan^2\theta \rightarrow$ a quadratic in $\tan\theta$: two angles or none
  - mango 10 m across, 5 m up, 14 m/s: $\tan\theta = 1$ or $3 \rightarrow 45°$ or 71.6°
- **off a height**: solve $y = -$(drop) for t
  - cliff 39.2 m, 19.6 m/s at 30°: 4 s, 67.9 m, 33.9 m/s at 60° below level
- thrown level: $\theta = 0$, fall time from $h = gt^2/2$
- at the same height twice: the two times are symmetric about the top
- any steady velocity one way + steady acceleration across $\rightarrow$ a parabola (an electron beam)

COMMON MISTAKES

✗ $F = ma$ for a changing mass
✓ $F = d(mv)/dt$.

✗ the tension = the hanging weight
✓ less: $T = m_1 m_2 g/(m_1 + m_2)$.

✗ the whole weight of a block on a slope pulls along it
✓ only $mg\sin\theta$.

✗ velocity at the top = 0
✓ horizontal, $u\cos\theta$.

✗ $R = u^2\sin^2\theta/g$
✓ $u^2\sin 2\theta/g$.

✗ greatest range at 90°
✓ at 45°; 90° goes straight up.

✗ one angle for a range
✓ θ and $90° - \theta$.

✗ from a cliff, $T = 2u\sin\theta/g$
✓ that is only for landing at the start's level; solve for t.