Unit 17 · Statics

STUDY NOTES

WHAT NEB'S GRADE 12 EXAM ASKS

- **CDC's grid (2078):** Mechanics (Units 17–18) **or** Computational Methods (Units 15–16), as the school teaches: 6 of 75 marks; 1 MCQ (an "OR" in the paper) and 1 short question; no long question
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours
- **CDC's model paper:** the short question split 3 + 2: one part on Newton's laws (Unit 18), one on forces; here, the tensions in two strings, a body on a smooth plane, or the angles between three forces; also two **like parallel forces** equivalent to one
- **MCQ (1 mark)**
  - Lami: each force $\propto$ what
  - forces along a triangle's sides in order
  - three equal forces: the angle
  - equal strings at 60°: the tension
  - $P : Q : R$ from three angles
  - smooth plane, horizontal force
  - which string is tighter
  - strings 3, 4, 5 m: a tension
  - force along a plane
  - forces parallel to a triangle's sides
  - the least holding force
- **Short and long (5 or 8 marks, in parts)**
  - strings at 30° and 45° to the horizontal
  - a plane, a force at 30° to it
  - angles from 1, 1, $\sqrt{3}$ N; like parallel forces
  - a string ABCD with two weights

Key terms

Equilibrium: the resultant of the forces at a point is zero.

Taken in order: head to tail round a triangle: AB, then BC, then CA.

Tension: a string's pull, along the string, away from the body.

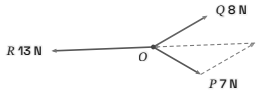
Reaction of a smooth plane: at right angles to the plane.

Like parallel forces: parallel, same direction; resultant their sum, between them.

Triangle law of forces

Statement: three forces at a point represented in magnitude and direction by the sides of a triangle taken in order are in equilibrium.

1. P, Q, R represented by $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$
2. $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$: the resultant of P, Q
3. R is $\overrightarrow{CA} = -\overrightarrow{AC}$: equal and opposite
4. total $\overrightarrow{AC} + \overrightarrow{CA} = 0$



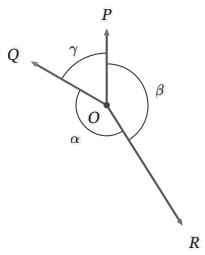
7, 8, 13 N in equilibrium; moved head to tail, they close a triangle.

- **converse** (used most): in equilibrium → they close a triangle, sides parallel and proportional to the forces
- **angle between two forces = 180° − the triangle's angle** between their sides
- 7, 8, 13 N: triangle angle opposite 13: $\cos C = -1/2$, 120° → the 7 and 8 N forces 60° apart
  - 8 and 13 N: 152.2°; 7 and 13 N: 147.8°
- three equal forces: an equilateral triangle → 120° apart

Lami's theorem

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

- α between Q and R; β between R and P; γ between P and Q: **the angle not touching the force**
- $\alpha + \beta + \gamma = 360°$



Each force faces the angle between the other two.

Proof:

1. the forces close a triangle, sides kP, kQ, kR
2. its angles are $180^\circ - \alpha$, $180^\circ - \beta$, $180^\circ - \gamma$
3. sine rule; $\sin(180^\circ - x) = \sin x$; cancel k

- only for **three** forces **in equilibrium** at a point; else resolve
- the largest force faces the angle nearest 90°
- **check**: $7/\sin 152.2^\circ = 8/\sin 147.8^\circ = 13/\sin 60^\circ = 15.01$

Strings and planes

$T_1 = \dfrac{W\sin\beta}{\sin(\alpha+\beta)}$	strings at α, β to the **vertical** **Use:** T_1 along α
$T_2 = \dfrac{W\sin\alpha}{\sin(\alpha+\beta)}$	the string nearer vertical is tighter **Use:** T_2 along β
$T = \dfrac{W}{2\cos\alpha}$	$\alpha = \beta$ **Use:** equal strings
$P = W\tan\theta$, $R = W\sec\theta$	smooth plane at θ **Use:** horizontal force

- **angles from the vertical**: the weight is straight down
  - 20 N, strings at 30° and 60°: angles 90°, 120°, 150° → $T_1 = 10\sqrt{3} \approx 17.32$ N, $T_2 = 10$ N
- angles given **to the horizontal**: turn them to the vertical first ($90^\circ -$ angle)
- **plane**: weight down; reaction ⊥ plane (θ from the vertical); the holding force
  - horizontal force, $\tan\theta = 3/4$, 50 N: $P = 37.5$ N, $R = 62.5$ N
  - along the plane: $P = W\sin\theta$
- string 3–4–5: a right angle at the knot
- a string with two weights: Lami at each knot; the level part BC carries the same tension both ways
  - horizontal parts of the end tensions are equal (nothing else pulls sideways)
- lantern at the middle, cord θ below level: $T = W/(2\sin\theta)$
  - $\theta \to 0$: T without limit; **no cord with a weight can be pulled straight**
- the least holding force: at right angles to the string

Like parallel forces

- $P + Q = R$; $P \times$ its distance from $R = Q \times$ its distance
- the resultant lies between them, nearer the larger
- 40 N, forces 2 m apart, one 0.5 m away: $0.5P = 1.5Q \to P = 30$ N, $Q = 10$ N

COMMON MISTAKES

✗ P paired with an angle touching P
✓ the angle between the other two.

✗ the angle between forces = the triangle's angle
✓ $180^\circ -$ it.

✗ angles to the horizontal used in Lami directly
✓ convert to angles between the forces.

✗ reaction of a smooth plane drawn vertical
✓ at right angles to the plane.

✗ Lami for four forces, or forces not in equilibrium
✓ three, in equilibrium; else resolve.

✗ a cord pulled hard enough becomes straight
✓ $T = W/(2\sin\theta)$ never allows $\theta = 0$.