Unit 16 · Linear Programming Problems

STUDY NOTES

WHAT NEB'S GRADE 12 EXAM ASKS

- **CDC's grid (2078):** Computational Methods (Units 15–16) **or** Mechanics (Units 17–18), as the school teaches: 6 of 75 marks; 1 MCQ and 1 short question; no long question
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours
- **CDC's model paper:** the short question split: 2 marks for Gauss–Seidel (Unit 15); then 3 marks, "using simplex method maximize Z ..." with two constraints
- **MCQ (1 mark)**
  - the pivot column
  - what a slack is
  - the pivot row from ratios
  - when to stop
  - why the smallest ratio
  - a basic feasible solution on the graph
  - read a final tableau
  - the first pivot entry
  - a whole small problem
  - a 0 under a non-basic variable
  - no positive entry: unbounded
- **Short (5 marks, in parts)**
  - maximize by simplex (3 marks)
  - maximize; name the corners passed
  - a cooperative's shawls and cloth: model, solve, shadow prices

Key terms

Objective function: the quantity to make largest, $Z = ax + by$.

Constraints: the limits, $2x + y \leq 16$; **feasible region**: the points keeping all of them.

Standard maximum problem: maximize Z; every constraint $\leq$ with right side ≥ 0; $x, y \geq 0$.

Slack variable: the amount of a resource left unused; ≥ 0.

Basic solution: two of the four variables set to 0 (**non-basic**), the other two solved (**basic**).

Basic feasible solution: a basic solution with nothing negative: a **corner**.

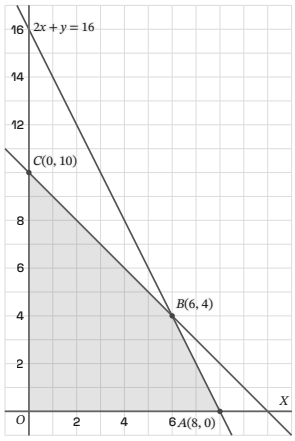
Tableau: the equations in a table, the objective row at the bottom.

Shadow price: what one more unit of a resource is worth.

Unbounded: Z can grow without end; no maximum.

Slack variables and corners

- $2x + y \leq 16 \to 2x + y + s_1 = 16$; $x + y \leq 10 \to x + y + s_2 = 10$
- feasible $\iff$ all four variables ≥ 0
- two equations, four unknowns: set two to 0, solve
- bakery
  - $x, y = 0$: O; $y, s_1 = 0$: $A(8, 0)$
  - $s_1, s_2 = 0$: $B(6, 4)$; $x, s_2 = 0$: $C(0, 10)$
  - $y, s_2 = 0$: $(10, 0)$, $s_1 = -4$: not feasible
- on a boundary line its slack (or x, or y) is 0; a corner: two such lines



The bakery: flour $2x + y \leq 16$, oven $x + y \leq 10$; corners O, A, B, C.

- many variables: millions of corners; simplex visits only a few, each better

The simplex method

First tableau ($Z - ax - by = 0$ at the bottom):

Basic	x	y	s_1	s_2	RHS
s_1	a_1	b_1	1	0	c_1
s_2	a_2	b_2	0	1	c_2
Z	$-a$	$-b$	0	0	0

1. **start at** O: slacks basic, $Z = 0$ (right sides ≥ 0: feasible)
2. **pivot column**: the most negative entry of the bottom row (raises Z fastest)
3. **pivot row**: the smallest RHS ÷ entry, over **positive** entries only (the first resource to run out)
4. **pivot**: divide the pivot row by the pivot; make the rest of its column 0 (the bottom row too); the column's variable comes in, the row's goes out
5. **repeat** until the bottom row has no negative entry
6. **read**: Z and the basic variables in the RHS column; non-basic = 0

- each tableau is one corner; each pivot moves along an edge to a better corner
- optimal: no edge leads higher; the region has no dents, so it is the best of all
- $Z = 4x - y$, $2x + y \leq 8$, $x - y \leq 1$: $(0, 0) \to (1, 0) \to (3, 2)$, $Z = 10$
- bakery $Z = 600x + 400y$: $O \to A(8, 0)$ (Rs 4,800) $\to B(6, 4)$, **Rs 5,200**
- check the answer in the objective and every constraint

Shadow prices:

- final bottom row $(0, 0, 200, 200 \mid 5200) \to Z = 5200 - 200s_1 - 200s_2$
- the entry under a slack: what one more unit of that resource is worth (flour Rs 200 a kg, oven Rs 200 an hour)
- a 0 under a non-basic variable: raising it leaves Z unchanged; the whole edge to the next corner ties (objective line parallel to it)

Traps

- ratio over **positive** entries only; skip 0 and negatives
- RHS ÷ the **pivot column's** entry
- the bottom row starts with the **negatives**: $Z = 4x - y \to -4, +1$
- no positive entry in the pivot column: **unbounded**
- two equal most-negative entries: either; same maximum

COMMON MISTAKES

✗ bottom row $(4, -1, \ldots)$ for $Z = 4x - y$
✓ $(-4, +1, \ldots)$: $Z - 4x + y = 0$.

✗ pivot column: the largest positive entry
✓ the most negative.

✗ pivot row: the largest ratio
✓ the smallest.

✗ a ratio with a negative or 0 entry used
✓ positive entries only.

✗ stopping while the bottom row has a negative entry
✓ continue until none.

✗ the non-basic variables read from the RHS
✓ they are 0.

✗ the bottom row left out of the elimination
✓ clear the pivot column there too.