Unit 11 · Probability

STUDY NOTES

WHAT NEB'S GRADE 12 EXAM ASKS

- **CDC's grid (2078):** Statistics and Probability (Units 10–11 together): 6 of 75 marks; 1 MCQ and 1 short question; no long question
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours
- **CDC's model paper:** the MCQ from probability
- **MCQ (1 mark)**
  - the definition of $P(B \mid A)$
  - $P(B \mid A)$ for independent events
  - two balls without replacement
  - $P(A \mid B)$ from three numbers
  - a die: given even
  - second card red, given the first
  - passed both, given one
  - three spades together
  - $P(A \mid B)$ from $P(A \cup B)$
  - two children, at least one boy
  - the second ball red
- **Short and long (5 or 8 marks, in parts)**
  - $P(A \mid B)$, $P(B \mid A)$, $P(\overline{A} \mid \overline{B})$; independent?
  - balls drawn together = one at a time; exactly two
  - a committee by lot: exactly, at least one, given at least one

Key terms

Dependent events: one's chance changes with the other's result (drawing without putting back).

Conditional probability $P(B \mid A)$: the chance of B given that A has happened.

Independent: knowing A doesn't change B's chance, $P(B \mid A) = P(B)$.

Selection: a group chosen all at once; every group of r from n equally likely.

Complement $\overline{B}$: B doesn't happen.

Formulas

$P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$	$P(A) > 0$ **Use:** conditional probability
$P(B \mid A) = \dfrac{n(A \cap B)}{n(A)}$	equally likely outcomes **Use:** count inside A
$P(A \cap B) = P(A)\,P(B \mid A)$	any two events **Use:** multiplication law
$P = \dfrac{\text{favorable selections}}{C(n, r)}$	groups chosen at once **Use:** by combinations

Three events, one factor a step:

$$P(A \cap B \cap C) = P(A)\,P(B \mid A)\,P(C \mid A \cap B)$$

From Grade 11: $P(E) = n(E)/n(S)$;
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$; independent $\iff P(A \cap B) = P(A)P(B)$

Dependent cases

- without putting back: each draw's chance **given the draws before it**
- tray of 12 eggs, 3 cracked
  - first cracked: 3/12
  - second, given the first cracked: 2/11; given whole: 3/11
  - both cracked: $(3/12)(2/11) = 1/22$
  - exactly one: cracked then whole **or** whole then cracked → add: 9/22
- independent events: the second factor is just $P(B)$ (Grade 11's law)
- the second ball red (3 red, 2 white): $(3/5)(2/4) + (2/5)(3/4) = 3/5$, as for the first

Conditional probability

- knowing A **shrinks the sample space to** A; count only the outcomes of A that are in B
- the formula ÷ $n(S)$ top and bottom → the definition (without proof)
- **properties**
  1. $0 \le P(B \mid A) \le 1$; $P(A \mid A) = 1$

2. $P(\overline{B} \mid A) = 1 - P(B \mid A)$
3. independent $\iff P(B \mid A) = P(B)$: the meaning behind $P(A \cap B) = P(A)P(B)$

- **$P(B \mid A)$ and $P(A \mid B)$ are different numbers**
  - two dice, A sum 8 (5 outcomes), B at least one 3 (11), $A \cap B$ 2
  - $P(B \mid A) = 2/5$; $P(A \mid B) = 2/11$
- from $P(A \cup B)$: first $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
- $P(\overline{A} \mid \overline{B}) = P(\overline{A} \cap \overline{B})/P(\overline{B})$
  - $= (1 - P(A \cup B))/(1 - P(B))$ (neither = not either)
- two children, given at least one boy: {BB, BG, GB} $\rightarrow$ both boys 1/3

Probability by combinations

- $n(S) = C(n, r)$; favorable groups by Unit 1's rules: **"and" multiplies, "or" adds**
- **"at least one"**: $1 - P(\text{none})$
- committee of 3 from 5 boys, 4 girls: $n(S) = C(9, 3) = 84$
  - exactly 2 girls: $C(4, 2)C(5, 1) = 30 \rightarrow 5/14$
  - at least one girl: $1 - C(5, 3)/84 = 37/42$
- **together = one at a time without replacement**
  - 20 bulbs, 4 faulty, test 3, none faulty: $C(16, 3)/C(20, 3) = (16/20)(15/19)(14/18) = 28/57$
- **given at least one**: exactly k lies inside "at least one", so
  - $P(\text{exactly } k \mid \text{at least one})$
  - $= P(\text{exactly } k)/P(\text{at least one})$

Large draws and small

- IPO lottery: $N \approx 19.6$ lakh applications, $k \approx 2.94$ lakh winners
  - none of a family of 4 wins: $C(N - 4, k)/C(N, k) \approx 0.522 \rightarrow$ at least one ≈ 0.478
  - son, given the mother won: $(k - 1)/(N - 1)$ against k/N: differ by less than one in a million
- from a **very large** number, nearly independent: $1 - 0.85^4 \approx 0.478$ too
- from a **small** number, not: 3 prizes in 30 tickets, one taken changes the next chance clearly

COMMON MISTAKES

✗ $P(B \mid A) = P(A \cap B)/P(B)$
✓ divide by $P(A)$, the event **given**.

✗ $P(B \mid A) = P(A \mid B)$
✓ different: count inside a different set.

✗ $P(A \cap B) = P(A)P(B)$ for draws without replacement
✓ $P(A)P(B \mid A)$.

✗ exactly one: only "cracked then whole"
✓ or "whole then cracked": add both orders.

✗ at least one counted case by case
✓ $1 - P(\text{none})$.

✗ drawn together: $P(n, r)$ for the count
✓ $C(n, r)$: a group has no order.

✗ the second draw's chance depends on an unknown first
✓ add the two cases; it equals the first draw's.