Unit 7 · Solution of Triangle

STUDY NOTES

WHAT NEB'S GRADE 12 EXAM ASKS

- **CDC's grid (2078):** Trigonometry (Units 6–7 together): 6 of 75 marks; with Analytic Geometry and Vectors, 3 MCQs, 2 short and 1 long question
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours
- **CDC's model paper:** MCQ: what kind of triangle has two given sides and the angle between them (solve it, then name it)
- **MCQ (1 mark)**
  - which data don't fix a triangle
  - the law for three sides
  - a third side from SAS
  - a side from a side and two angles
  - an angle from three sides
  - how many triangles fit
  - width of a river from two sightings
  - kind of triangle from SAS
  - largest angle of sides in x
- **Short and long (5 or 8 marks, in parts)**
  - solve from three sides (surds)
  - solve from two sides and the angle between them
  - distances to a far point from a base and two angles; a river's width
  - a hilltop: distances, then height by the angle of elevation; effect of a 1° error

Key terms

Solving a triangle: finding all six parts (three sides, three angles) from those given.

Fixes a triangle: SSS, SAS, ASA (or AAS), RHS, as the congruence tests.

Three angles: shape only, never size.

Two sides and an angle not between them: may fit two triangles, one or none.

Which law for which data

Given	First	Then
three sides (SSS)	cosine law: the **largest** angle	cosine law again, then 180° − the two
two sides, angle between (SAS)	cosine law: third side	sine law for the **smaller** angle; then 180° −
a side, two angles (ASA, AAS)	third angle = 180° − the two	sine law for each side
two sides, angle opposite the **longer**	sine law: the other angle (acute)	180° −; sine law for the third side

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$	one angle, 0°–180° **Use:** from three sides
$a^2 = b^2 + c^2 - 2bc\cos A$	A between b, c **Use:** third side
$b = \dfrac{a \sin B}{\sin A}$	$A = 180° - B - C$ **Use:** sides from angles
$\sin B = \dfrac{b \sin A}{a}$	only if B is known acute **Use:** an angle by the sine law

The sine law's trap

- $\sin^{-1}$ gives only −90° to 90°; θ and $180° - \theta$ have the same sine
  - $\sin B = 0.5$: 30° **or** 150°
- $\cos^{-1}$ gives one angle, 0°–180°: never in doubt
- so by the sine law find only an angle **opposite a side that isn't the longest**: it is acute
- the largest angle faces the longest side; only it can be obtuse
- $b = 2$, $c = \sqrt{3} + 1$, $A = 60°$: $a = \sqrt{6}$; B (shortest side) = 45°; $C = 75°$
  - C by the sine law would allow 75° or 105°

How many triangles? (two sides, an angle not between)

Given a, b and A (opposite a): the side a swings about C; the nearest it can reach to A's other arm is $h = b \sin A$.

A acute and	Triangles
$a < b \sin A$	none ($\sin B > 1$)
$a = b \sin A$	one, right-angled at B
$b \sin A < a < b$	two: B and $180^\circ - B$
$a \geq b$	one

- A right or obtuse: one if $a > b$, none otherwise
- $a = 4, b = 3, A = 50^\circ$: $a > b \to$ one
- $a = 6, b = 8, A = 30^\circ$: $h = 4 < 6 < 8 \to$ two ($B \approx 41.81^\circ$ or 138.19°)
- NEB asks only $a \geq b$; the two-triangle case is for the engineering entrance

Checks and examples

- check: $a/\sin A = b/\sin B = c/\sin C$ (5, 6, 7: each ≈ 7.14)
- 5, 6, 7: $\cos C = 0.2$, $C \approx 78.46^\circ$; $\cos A = 5/7$, $A \approx 44.42^\circ$; $B \approx 57.12^\circ$
- $a = 10, B = 45^\circ, C = 75^\circ$: $A = 60^\circ$, $b \approx 8.16$, $c \approx 11.15$
- exact values: $\sin 75^\circ = (\sqrt{6} + \sqrt{2})/4$, $\cos 120^\circ = -1/2$
- **kind of triangle** from SAS: find the third side (or angles), then compare
  - $b = 2, c = 1, A = 60^\circ$: $a = \sqrt{3}$, $a^2 + c^2 = b^2 \to$ right-angled at B

Distances by angles (triangulation)

- a measured **base** and the angle at each end fix any far point (ASA)
  - distance: sine law; the angle at the far point = 180° − the two
  - river width = (distance to the tree) × sin (the angle at that end)
- **chain of triangles**: each new side the base of the next; after the first base, angles only
- a slip in one angle passes into every later triangle $\to$ measure angles many times
- a height: horizontal distance × tan (angle of elevation)

COMMON MISTAKES

✗ an angle facing the longest side found by $\sin^{-1}$
✓ it may be obtuse: use the cosine law.

✗ three angles called enough
✓ they fix the shape, not the size.

✗ SAS: the sine law first
✓ no angle faces a known side yet: cosine law for the third side.

✗ the river's width = PA
✓ the perpendicular: $PA \sin A$.

✗ the third angle by the sine law
✓ 180° − the other two: no doubt.