Unit 2 · The Limits of Computing

WORKSHEET

Full marks 34

Name _____ Class _____ Roll no. ____ Date _____ Marks _____ / 34

1 Key words [4]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

accepting states	deterministic	finite state machine	grammar	NP-complete	polynomial
recursion	space complexity	Turing complete	undecidable		

How the extra memory a program needs grows with its data is its _____. An algorithm that takes exactly the same steps every time for the same input is _____. A machine with a fixed number of states that moves between them on its inputs is a _____, and when it reads a text, its double circles are its _____. A set of rules that can use themselves, written in BNF, is a _____. A language that can do everything a Turing machine can is called _____. The halting problem can't be solved by any program: it is _____. A problem in NP that every problem in NP can be turned into quickly is _____.

2 Multiple choice [5]

Choose the correct option for each of the following questions.

i. Which is the big O of a program that does $4n^2 + 100n + 7$ steps? [1]

a. $O(4n^2)$ **b.** $O(100n)$
c. $O(n^2)$ **d.** $O(n^2 + n)$

ii. A program reads n exam results and keeps them all in a dictionary, by roll number, so it can answer questions later. What is its extra memory? [1]

a. O(1): a dictionary is one variable **b.** O(log n)
c. O(n): one entry for each result **d.** $O(n^2)$

iii. What is `6 2 ÷ 1 +` in ordinary notation, and its value? [1]

a. $6 \div (2 + 1) = 2$ **b.** $6 \div 2 + 1 = 4$
c. $2 \div 6 + 1$ **d.** $6 \div 2 \div 1 = 3$

iv. What does a Turing machine have that a finite state machine doesn't? [1]

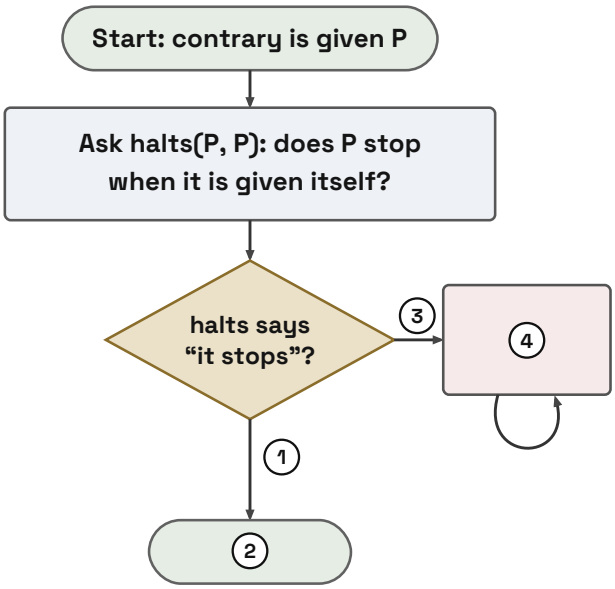
a. states
b. rules that depend on the symbol read
c. a tape it can write on and read back, as long as it needs
d. a start state

v. A problem is in NP. What does that tell you for certain? [1]

a. There is no fast way to solve it.
b. When the answer is yes, a proposed answer can be checked quickly.
c. It is NP-complete.
d. It can't be solved at all.

3 The halting problem [2]

The diagram shows what the program `contrary` does with a program P. Write the word or words that belong in place of 1 to 4. Each carries 0.5 mark.



1. _____ 2. _____

3. _____ 4. _____

4 Will it finish in time? [3]

Answer on the lines. Show your working.

A program puts the SEE results of 5,000 students in order of marks in 2 seconds.

i. If the program is $O(n^2)$, how long will it take for 20,000 students, and for the whole country's 500,000?

ii. If it is O(n log n) instead, about how long for 20,000 students? ($\log_2 5{,}000 \approx 12.3$ and $\log_2 20{,}000 \approx 14.3$.)

5 Think [2]

Think, then answer.

Sita says: "Computers get faster every year, so one day they will be able to tell whether any program stops, and find the shortest trek through any number of villages." Is she right about each? Explain.

6 A machine for class names [4]

A class is named by its grade, one or two digits, then one capital letter: `9B` and `10A` are class names. The pattern is `\d\d?[A-Z]`, and this machine checks it: it starts in S, A means one digit read, B two digits read, and C, the only accepting state, a letter read after them.

Fill in the transition table. Write – where the machine has no arrow, so the text is rejected.

State	a digit	a capital letter
S		
A		
B		
C		

i. Which of these does the machine accept: `10A`, `123A`, `9`, `B9`? For each one it rejects, say where it stops.

7 The shunting-yard algorithm [4]

Turn the expression into RPN with the shunting-yard algorithm. Write the output and the operator stack (bottom first) after each symbol. Then answer.

`7 - 2 × (4 - 1)`

Symbol	Output	Operator stack
7		
-		
2		
×		
(		
4		
-		
1		
)		
end		

i. Write the RPN, and work it out with a stack.

8 A Turing machine adds one [3]

The unit's machine that adds one in binary starts in state right, with its head on the first digit of `1001`. Its rules:

State	Reads	Writes	Moves	Next state
right	0	0	R	right
right	1	1	R	right
right	blank	blank	L	carry
carry	1	0	L	carry
carry	0	1	stay	done
carry	blank	1	stay	done

Write one row for each step until the machine stops in done.

Step	State	Reads	Writes	Moves	Next state

i. What is on the tape at the end? What sum has the machine done, in ordinary numbers?

9 Write a program [4]

Write the program in Python on the lines. Any program that gives the right result gets the marks.

Write a function `is_class_name(text)` that runs your machine of question 6 on a text, one character at a time, and gives back True if it accepts the text and False if not. Keep the machine's arrows in a dictionary, as the unit does. Then use it on `"10A"`, `"123A"` and `"9"`.

10 Find and fix the bug [3]

Read the program, then answer.

The program should work out RPN with a stack, as in the unit. It prints `11.0`, then `7.0` and `0.4`, but `8 3 - 2 -` is 3 and `10 4 /` is 2.5.

```
def rpn(text):
    stack = []
    for token in text.split():
        if token in ("+", "-", "*", "/"):
            left = stack.pop()
            right = stack.pop()
            if token == "+":
                stack.append(left + right)
            elif token == "-":
                stack.append(left - right)
            elif token == "*":
                stack.append(left * right)
            else:
                stack.append(left / right)
        else:
            stack.append(float(token))
    return stack[0]

print(rpn("3 4 2 * +"))
print(rpn("8 3 - 2 -"))
print(rpn("10 4 /"))
```

i. What is wrong, and why is the first answer right anyway?

ii. Write the corrected lines.

11 Activity: The pattern as a machine (not marked)

At a computer or on a phone, open the unit's page (the QR code) and find **The pattern as a machine**, the machine for an amount in rupees, `\d+(\.\d\d)?`. Read each text through it and fill in the table.

Text	Accepted or rejected?	Where it ended or stopped
12.50		
12.5		
.50		
1,200		
१२५		