Unit 2 · The Limits of Computing

ANSWER KEY · FOR TEACHERS

Full marks 34

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. space complexity
2. deterministic
3. finite state machine
4. accepting states
5. grammar
6. Turing complete
7. undecidable
8. NP-complete

Each carries 0.5 mark. **K · 4**

2 Multiple choice

i. **c.** $O(n^2)$ **K · 1**

Keep the fastest-growing part and drop the number in front: n^2. For big n, 100n is tiny beside $4n^2$, and big O is about how the work grows, so the 4 doesn't matter either.

ii. **c.** O(n): one entry for each result **U · 1**

Each result adds one entry, so the dictionary grows in step with n. That memory buys speed: every later lookup is O(1).

iii. **b.** $6 \div 2 + 1 = 4$ **A · 1**

6 and 2 are pushed; ÷ pops them and pushes $6 \div 2 = 3$ (the first number pushed is on the left); 1 is pushed; + pops 1 and 3 and pushes 4. For $6 \div (2 + 1)$, the RPN would be 6 2 1 + ÷.

iv. **c.** a tape it can write on and read back, as long as it needs **K · 1**

Both have a fixed list of states and rules. The tape is a memory that never runs out: the brackets machine kept its count on the tape by crossing brackets off, which no fixed set of states can do for every depth.

v. **b.** When the answer is yes, a proposed answer can be checked quickly. **U · 1**

NP means "quick to check". Every problem in P is in NP too, so being in NP doesn't mean being hard. Being unsolvable is the halting problem's kind of trouble, which is a different thing: NP problems can always be solved by trying every possible answer, however slowly.

3 The halting problem

1. No
2. Stop
3. Yes
4. Loop forever

Each carries 0.5 mark. **U · 2**

4 Will it finish in time?

MARKING SCHEME **A · 3**

- ✓ 32 seconds for 20,000 students ($4^2 = 16$ times).
- ✓ About 20,000 seconds (5.6 hours) for 500,000 ($100^2 = 10{,}000$ times).
- ✓ About 9 seconds for O(n log n) ($4 \times 14.3 \div 12.3 \approx 4.65$ times).

5 Think

ANSWER

She is wrong about both, for different reasons. The halting problem is **undecidable:** Turing proved that no program can answer it for every program, however fast the computer,

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because a program built from the checker does the opposite of what it says. The shortest trek can always be found by trying every route, but the work grows like n!, so a computer a thousand times faster manages only a few more villages; it is NP-complete, and unless P = NP (which most experts doubt) there is no fast method to find. HA · 2

MARKING SCHEME

- ✓ The halting problem can't be solved by any program at all, whatever the speed (proved, not a matter of speed).
- ✓ The trek can be solved, but its work grows so fast that speed doesn't help much; a fast method exists only if P = NP.

6 A machine for class names

ANSWER A · 4

State	a digit	a capital letter
S	A	–
A	B	C
B	–	C
C	–	–

i. Only `10A` is accepted: S → A → B → C. `123A` is rejected at the third digit (B has no arrow for a digit). `9` ends in A, which is not accepting: the text stopped too soon. `B9` is rejected at once: S has no arrow for a letter.

MARKING SCHEME

- ✓ S and A rows correct (S: A, –; A: B, C).
- ✓ B and C rows correct (B: –, C; C: –, –).
- ✓ `10A` accepted, the others rejected.
- ✓ Where each rejection happens: no arrow (`123A`, `B9`) or ending in a state that isn't accepting (`9`).

7 The shunting-yard algorithm

ANSWER A · 4

Symbol	Output	Operator stack
7	7	
−	7	−
2	7 2	−
×	7 2	− ×
(	7 2	− × (
4	7 2 4	− × (
−	7 2 4 1	− × (−
1	7 2 4 1	− × (−
)	7 2 4 1 −	− ×
end	7 2 4 1 − × −	

i. RPN: **7 2 4 1 − × −**. With a stack: 7, 2, 4, 1 pushed; − pops 1 and 4 and pushes 3; × pops 3 and 2 and pushes 6; − pops 6 and 7 and pushes 7 − 6 = **1**.

MARKING SCHEME

- ✓ × goes onto the stack on top of − (− binds less tightly), and (on top of both.
- ✓ At), the − inside the brackets goes to the output and the (is thrown away.
- ✓ At the end, × then − leave the stack: 7 2 4 1 − × −.
- ✓ Worked out with a stack: 1.

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8 A Turing machine adds one

ANSWER **A · 3**

Step	State	Reads	Writes	Moves	Next state
1	right	1	1	R	right
2	right	0	0	R	right
3	right	0	0	R	right
4	right	1	1	R	right
5	right	blank	blank	L	carry
6	carry	1	0	L	carry
7	carry	0	1	stay	done

i. The tape reads **1010**, after 7 steps: 1001 is nine, and 1010 is ten, so it has worked out 9 + 1 = **10**.

MARKING SCHEME

- ✓ Steps 1 to 5: right over the four digits, then back onto the last digit in carry at the blank.
- ✓ Steps 6 and 7: the last 1 becomes 0, the 0 before it becomes 1, and the machine stops in done.
- ✓ 1010: nine plus one is ten.

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9 Write a program

ANSWER **A · 4**

```
NEXT = {
    "S": {"digit": "A"},
    "A": {"digit": "B", "capital": "C"},
    "B": {"capital": "C"},
    "C": {},
}

def kind(ch):
    if ch.isdecimal():
        return "digit"
    if "A" <= ch <= "Z":
        return "capital"
    return "other"

def is_class_name(text):
    state = "S"
    for ch in text:
        state = NEXT[state].get(kind(ch))
        if state is None:
            return False            # no arrow: rejected
    return state == "C"

for t in ["10A", "123A", "9"]:
    print(t, is_class_name(t))
```

```
OUTPUT
10A True
123A False
9 False
```

MARKING SCHEME

- ✓ The arrows kept as data (a dictionary of states, or of (state, kind) pairs).
- ✓ Each character sorted into digit, capital letter or other.
- ✓ No arrow means False at once; otherwise the state moves on.
- ✓ True only if it ends in C, and used on the three texts.

10 Find and fix the bug

MARKING SCHEME **A · 3**

- ✓ The first number popped is the right-hand one, but the program takes it as the left.
- ✓ + and × don't care about the order, so only − and ÷ go wrong.
- ✓ `right = stack.pop()` first, then `left = stack.pop()`.

11 Activity: The pattern as a machine

(not marked)

WHAT STUDENTS SHOULD FIND

- `12.50` ends in D, a double circle: accepted.
- `12.5` ends in C, still waiting for a second paisa digit: rejected.
- `.50` stops at once in S (no arrow for a dot), and `1,200` stops in A at the comma: rejected.
- `१२५` is accepted, ending in A: Nepali digits are digits too.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	6	4	22	2	**34**

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