Unit 18 · Dispersion

ANSWER KEY · FOR TEACHERS

Full marks 37

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. dispersion
2. violet
3. spectrum
4. pure
5. angular dispersion
6. chromatic aberration
7. spherical aberration
8. achromatic doublet

K · 4

Each carries 0.5 mark.

2 Label the diagram

1. violet, 40.6°
2. red, 42.4°
3. reflects
4. raindrop

U · 2

Each carries 0.5 mark.

3 Multiple choice

i. **d.** violet — K · 1

Glass has the largest refractive index for violet (the shortest wavelength), so $\delta = (\mu - 1)A$ is largest for violet.

ii. **b.** air molecules scatter blue light much more than red — U · 1

Rayleigh scattering, $I \propto 1/\lambda^4$: blue is scattered about 6 times as strongly as red. No refraction splits the light into a spectrum, so it is not dispersion.

iii. **a.** 42° from the point opposite the Sun, violet at 40° — U · 1

Red comes from drops 42° from the antisolar point, violet from drops at 40°, a slightly smaller circle inside.

iv. **b.** its material only (for the two colors compared) — K · 1

$\omega = (\mu_v - \mu_r)/(\mu - 1)$ has no A in it: a property of the material alone.

v. **c.** a convex and a concave lens of different glasses — U · 1

From $\omega/f + \omega'/f' = 0$, f and f′ have opposite signs, and the glasses must differ, or the pair would have no power.

4 Very short answers

a. MARKING SCHEME — K · 1

✓ $\omega = (\delta_v - \delta_r)/\delta$, angular dispersion ÷ mean deviation.

b. MARKING SCHEME — K · 1

✓ $\mu = A + B/\lambda^2 + C/\lambda^4 + \ldots$ (A, B, C constants).

c. MARKING SCHEME — U · 1

✓ Yellow is deviated by about the average of all the colors' deviations.

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5 Short answers

a. K · 2

MARKING SCHEME

- ✓ Any one, e.g. a stop that lets only paraxial rays through.
- ✓ Any second, e.g. sharing the bending between the faces (curved face to the parallel light), a convex and a concave lens whose aberrations cancel, or aspheric surfaces.

b. U · 2

MARKING SCHEME

- ✓ $I \propto 1/\lambda^4$, so the ratio is $(700/450)^4 \approx 6$.
- ✓ Sunlight has less violet than blue, and our eyes are much less sensitive to violet.

c. K · 2

MARKING SCHEME

- ✓ Any one, e.g. a narrow slit, or a parallel beam falling on the prism (slit at the focus of the collimator lens).
- ✓ Any second, e.g. the prism at minimum deviation, or the colors focused by an achromatic lens.

6 Long answer

U · 4

ANSWER

By the lens maker's formula for the mean color, $1/f = (\mu - 1)(1/R_1 + 1/R_2)$, so $(1/R_1 + 1/R_2) = 1/(f(\mu - 1))$. The radii are the same for every color, so $1/f_r = (\mu_r - 1)/(f(\mu - 1))$ and $1/f_v = (\mu_v - 1)/(f(\mu - 1))$. Subtracting, $1/f_v - 1/f_r = (\mu_v - \mu_r)/(f(\mu - 1)) = \omega/f$, that is $(f_r - f_v)/(f_r f_v) = \omega/f$. The two focal lengths differ by only a few percent, so $f_r f_v \approx f^2$, and $f_r - f_v = \omega f$.

MARKING SCHEME

- ✓ $1/R_1 + 1/R_2 = 1/(f(\mu - 1))$ from the mean color.
- ✓ $1/f_r$ and $1/f_v$ written with μ_r and μ_v.
- ✓ $1/f_v - 1/f_r = \omega/f$, so $(f_r - f_v)/(f_r f_v) = \omega/f$.
- ✓ $f_r f_v \approx f^2$, giving $f_r - f_v = \omega f$.

7 Numerical problems

a. A · 3

ANSWER

$\delta_v = 0.530 \times 6^\circ = 3.180^\circ$, $\delta_r = 0.514 \times 6^\circ = 3.084^\circ$, so the angular dispersion is $\theta = \mathbf{0.096}^\circ$.
δ (yellow) $= 0.517 \times 6^\circ = 3.102^\circ$, so $\omega = 0.096 \div 3.102 = \mathbf{0.031}$.

MARKING SCHEME

- ✓ $\delta_v = 3.180^\circ$ and $\delta_r = 3.084^\circ$.
- ✓ $\theta = 0.096^\circ$.
- ✓ $\omega = 0.031$.

b. A · 3

ANSWER

$f_r - f_v = \omega f = 0.031 \times 40 = \mathbf{1.24\ cm}$.
f_r is larger, so **violet** is focused nearer the lens (about 1.2 cm nearer than red).

MARKING SCHEME

- ✓ $f_r - f_v = \omega f$.
- ✓ 1.24 cm.
- ✓ Violet is focused nearer the lens.

c. A · 4

ANSWER

$\omega_1/f_1 + \omega_2/f_2 = 0$, so $f_2 = -(\omega_2/\omega_1)f_1 = -2.44 f_1$.
$1/F = 1/f_1 + 1/f_2 = (1/f_1)(1 - 1/2.44) = 0.590/f_1 = 1/50$, so $f_1 = \mathbf{+29.5\ cm}$ (crown, convex) and $f_2 = -2.44 \times 29.5 = \mathbf{-71.9\ cm}$ (flint, concave).

MARKING SCHEME

- ✓ The condition $\omega_1/f_1 + \omega_2/f_2 = 0$, giving $f_2 = -2.44 f_1$.
- ✓ $1/F = 1/f_1 + 1/f_2$ with F = 50 cm.
- ✓ $f_1 = +29.5$ cm, the crown lens, convex.
- ✓ $f_2 = -71.9$ cm, the flint lens, concave.

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8 Think

ANSWER HA · 3

The bow is a circle of radius about 42° round the point directly opposite the Sun (the shadow of your head), which is as far below the horizon as the Sun is above it. With the Sun higher than 42°, that point is more than 42° below the horizon, so the whole circle is below the horizon: there is no bow to see. From an aeroplane there are drops below the horizon too, so the whole circle can be seen.

MARKING SCHEME

- ✓ The bow is a circle of about 42° round the point opposite the Sun, below the horizon by the Sun's height.
- ✓ With the Sun above 42°, the whole circle lies below the horizon (on the ground), so no bow is seen.
- ✓ From an aeroplane there are drops in every direction round that point, so the full circle shows.

9 Activity: A spectrum from a glass of water

(not marked)

WHAT STUDENTS SHOULD FIND

Red at one end and violet at the other, in the order red, orange, yellow, green, blue, violet; violet is bent farthest from the straight path (the largest refractive index). Farther away the band is wider and fainter. The colors blend into each other with no edges and overlap: it is an impure spectrum, since the light from the window and the Sun's disc is not a narrow parallel beam and nothing focuses each color.

See it at Science Playground

Exhibits and plants at the center, Bhaktapur, that go with this unit. Book a class visit at science-playground.com/booking.

- **String Prism** (exhibit): The strings show white light fanning into colors in the prism, red bent least and violet most, since glass's refractive index is largest for violet.
- **The Spectroscope** (exhibit): Look into the viewing window: white light spreads into a band of colors, its spectrum, and different light sources show slightly different patterns.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	12	12	10	3	**37**

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