Unit 13 · Ideal Gas

ANSWER KEY · FOR TEACHERS

Full marks 32

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. ideal gas
2. gas constant
3. mole
4. Avogadro's number
5. Avogadro's law
6. rms speed
7. Boltzmann's constant
8. absolute zero

K · 4

Each carries 0.5 mark.

2 True or false

i. ✓ **True**

ii. ✗ **False**: V ∝ T only in kelvin: doubling the absolute temperature doubles the volume.

iii. ✗ **False**: It is proportional to the square root of the absolute temperature, $c_{rms} = \sqrt{3RT/M}$.

iv. ✓ **True**

U · 2

Each carries 0.5 mark.

3 Multiple choice

i. **b.** J/(mol K) — K · 1

R = PV/(nT): J ÷ (mol × K), so J/(mol K). Its value is 8.31 J/(mol K).

ii. **d.** the molecules attract each other strongly — U · 1

An ideal gas has no forces between its molecules except at collisions; strong attraction is what makes real gases depart from ideal behaviour.

iii. **c.** 44.8 L — A · 1

One mole takes 22.4 L at NTP, so 2 moles take 44.8 L.

iv. **c.** 327 °C — A · 1

P ∝ T in kelvin: 300 K → 600 K, which is 327 °C.

v. **c.** the absolute temperature — U · 1

$\frac{1}{2}mc_{rms}^2 = (3/2)k_B T$: it depends on T only, the same for every gas at the same temperature.

4 Short answers

a. MARKING SCHEME — U · 2

✓ Low pressure: molecules far apart, so their volume and the forces between them are negligible.

✓ High temperature: fast molecules, so the attractions matter little.

b. MARKING SCHEME — U · 2

✓ Heating makes the molecules move faster (KE ∝ T).

✓ More frequent and harder collisions raise the pressure, so the balloon expands.

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c. MARKING SCHEME A · 3

- ✓ n = PV/(RT), with V in m^3 and T in K.
- ✓ n = 0.80 mol.
- ✓ Mass 25.7 g.

5 Long answer

ANSWER U · 4

A molecule with velocity component u towards the wall X bounces back elastically with −u: its momentum changes by 2mu. It returns to the same wall after travelling 2l, in time 2l/u, so it hits that wall u/(2l) times a second. The force it exerts is the rate of change of momentum: 2mu × u/(2l) = mu^2/l.

For all N molecules, the force on the wall is (m/l)($u_1^2 + u_2^2 + \ldots$), and the pressure is force ÷ l^2:

$$P = \frac{m}{l^3}(u_1^2 + u_2^2 + \ldots) = \frac{mN}{V}\overline{u^2}.$$

The motion is random, so $\overline{u^2} = \overline{v^2} = \overline{w^2} = \frac{1}{3}\,crms^2$.

So $P = \frac{mN}{3V}crms^2 = \frac{1}{3}\rho crms^2$, since mN/V is the density ρ.

MARKING SCHEME

- ✓ Momentum change 2mu per bounce, bounces u/(2l) a second, force mu^2/l.
- ✓ Total pressure P = (mN/V) times the mean of u^2.
- ✓ Random motion: mean of $u^2 = ⅓c_{rms}^2$.
- ✓ $P = ⅓\rho c_{rms}^2$.

6 Numerical problems

a. ANSWER A · 4

i. $$crms = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 300}{0.028}} = \mathbf{517\ m/s}.$$

ii. $$\frac{1}{2}\,mcrms^2 = \frac{3}{2}\,k_B T = 1.5 \times 1.38{\times}10^{-23} \times 300 = \mathbf{6.21 \times 10^{-21}\ J}.$$

MARKING SCHEME

- ✓ $c_{rms} = \sqrt{(3RT/M)}$ with M in kg/mol and T in K.
- ✓ 517 m/s.
- ✓ KE = (3/2)$k_B T$.
- ✓ 6.21×10^{-21} J.

b. ANSWER A · 3

$$n = \frac{PV}{RT} = \frac{1.5{\times}10^7 \times 0.020}{8.31 \times 288} = 125 \text{ mol}.$$

Mass = 125 × 0.032 = **4.0 kg**.

MARKING SCHEME

- ✓ n = PV/(RT) with T = 288 K.
- ✓ n = 125 mol.
- ✓ Mass 4.0 kg.

7 Think

ANSWER HA · 3

Each perfume molecule does move very fast, but the room is full of air molecules. It collides with them billions of times a second, and every collision sends it off in a new random direction. So its path is a zigzag of tiny steps, and it makes very slow progress in any one direction, though its speed between collisions is high.

MARKING SCHEME

- ✓ The molecules are fast, but collide with air molecules very often.
- ✓ Each collision changes the direction at random.
- ✓ So the zigzag path makes slow progress across the room.

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8 Activity: A balloon on a bottle

(not marked)

WHAT STUDENTS SHOULD FIND

In hot water the balloon fills and stands up; in cold water it is sucked in, partly into the bottle. Warmed, the air molecules move faster and hit the walls harder, so the pressure rises and the air expands into the balloon ($V \propto T$ at constant pressure, T in kelvin). Cooled, they slow, the pressure inside falls, and the outside air pushes the balloon in.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	5	12	12	3	**32**

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