Unit 7 · Gravitation

HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 29

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Spot the error

ANSWER U · 3

g(1 − 2h/R) is an approximation that holds only when h is much smaller than R. Here h = R/2, so the exact formula is needed: $g' = gR^2/(R + h)^2 = 9.8 \times (6400 \div 9600)^2 =$ **4.36 m/s²**, nearly half of g. Gravity is far from zero there.

MARKING SCHEME

- ✓ The approximate formula is used where h is not small compared with R.
- ✓ The exact $g' = gR^2/(R + h)^2$.
- ✓ $g' = 4.36$ m/s².

2 Predict, then explain

ANSWER A · 4

i. **Sanjay's**, going up.

ii. Below the surface: g′ = g(1 − d/R) = g(1 − 100/6400) = **0.984g**. Above: g′ ≈ g(1 − 2h/R) = g(1 − 200/6400) = **0.969g**. For small distances, g falls twice as fast going up as going down: going down, the shell of rock above stops pulling, but the mass still below is closer.

MARKING SCHEME

- ✓ Sanjay's weight drops more.
- ✓ Down: 1 − d/R = 0.984.
- ✓ Up: 1 − 2h/R = 0.969.
- ✓ g falls about twice as fast with height as with depth.

3 Read the data

ANSWER A · 4

i. (R + h) = 6400, 7040, 9600, 12 800 km. $g(R + h)^2$: $9.80 \times 6400^2 = 4.01 \times 10^8$; $8.10 \times 7040^2 = 4.01 \times 10^8$; $4.36 \times 9600^2 = 4.02 \times 10^8$; $2.45 \times 12\,800^2 = 4.01 \times 10^8$ (km² m/s²): the same, so $g \propto 1/(R + h)^2$.

ii. The first 640 km lowers g by 1.70 m/s². At that rate 6400 km would take away 17 m/s², more than g itself, but the table shows a drop of only 7.35 m/s²: g falls ever more slowly as you go up.

iii. R + h = 19 200 km = 3R, so g′ = g/9 = **1.09 m/s²**.

MARKING SCHEME

- ✓ $g(R + h)^2$ worked out and found the same (4.0×10^8).
- ✓ So $g \propto 1/(R + h)^2$.
- ✓ Ayush is wrong: a steady drop of 1.7 per 640 km doesn't fit the table.
- ✓ 1.09 m/s² at 3R from the center.

4 Launch costs

ANSWER A · 4

i. In orbit: $E = -GMm/(2 \times 2R) = -mgR/4 = -1000 \times 9.8 \times 6.4 \times 10^6 \div 4 =$ **-1.57×10^{10} J**. On the ground: $U = -GMm/R = -mgR =$ **-6.27×10^{10} J**.

ii. Energy needed = E − U = −mgR/4 + mgR = ¾mgR = **4.70×10^{10} J**.

MARKING SCHEME

- ✓ E in orbit = $-GMm/4R = -1.57 \times 10^{10}$ J.
- ✓ U on the ground = $-mgR = -6.27 \times 10^{10}$ J.
- ✓ Energy needed = E − U = ¾mgR.
- ✓ 4.70×10^{10} J.

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5 Show that

ANSWER A · 3

Orbiting at r = R: $\frac{GMm}{R^2} = \frac{mv_o^2}{R}$, so $v_o = \sqrt{GM/R}$.

Escaping: $\frac{1}{2}mv_e^2 = \frac{GMm}{R}$, so $v_e = \sqrt{2GM/R} = \sqrt{2}\,v_o$ (11.2 km/s and 7.9 km/s).

$\sqrt{2} \approx 1.41$, so 41% more speed is the escape velocity: it would leave the Earth for good.

MARKING SCHEME

- ✓ $v_o = \sqrt{(GM/R)}$.
- ✓ $v_e = \sqrt{(2GM/R)} = \sqrt{2}\,v_o$.
- ✓ 41% more is the escape velocity: it escapes.

6 Estimate it

ANSWER HA · 3

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3g}{4\pi GR} = \frac{3 \times 9.8}{4\pi \times 6.67{\times}10^{-11} \times 6.4{\times}10^{6}} \approx \mathbf{5500\ kg/m^3}.$$

That is about twice the density of surface rock, so the inside must be much denser: the Earth has a dense core (mostly iron).

MARKING SCHEME

- ✓ $\rho = 3g/(4\pi GR)$.
- ✓ About 5500 kg/m^3.
- ✓ Twice the surface rock: a dense core inside.

7 Correct the misconceptions

ANSWER U · 3

1. Orbital speed, $\sqrt{(GM/r)}$, doesn't depend on the satellite's mass: heavy and light satellites in the same orbit move at the same speed.
2. At the center g is zero: the pulls of the mass all around cancel ($g' = g(1 - d/R) = 0$ at $d = R$).
3. With zero at infinity, gravitational potential energy is negative everywhere, $U = -GMm/r$; higher means less negative, not positive.

MARKING SCHEME

- ✓ Orbital speed is the same for any mass.
- ✓ g is zero at the center.
- ✓ Potential energy is negative (zero only at infinity).

8 Think

ANSWER HA · 3

Drag takes energy away, so the total energy $E = -GMm/2r$ becomes more negative, which means a smaller r: the satellite sinks. But its kinetic energy is $K = GMm/2r = -E$, so as E falls K rises: it moves faster. The potential energy, $U = -GMm/r$, falls by twice as much as K rises; half of that loss becomes kinetic energy and half goes to the drag (heat).

MARKING SCHEME

- ✓ Drag lowers E, so r gets smaller (it sinks).
- ✓ $K = -E$, so K and the speed increase.
- ✓ U falls by twice the gain in K; the rest is lost to the drag.

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9 Make your own

ANSWER HA · 2

For example, a planet with 2 times the Earth's mass and 2 times its radius: $v_e = \sqrt{2GM/R}$ is unchanged (M/R the same), but $g = GM/R^2$ is 2/4 = **half** the Earth's, 4.9 m/s^2. (Any mass and radius in the same ratio.)

MARKING SCHEME

- ✓ A planet with M/R the same as the Earth's.
- ✓ Its g found (e.g. g/2 for 2M, 2R).

See it at Science Playground

Exhibits and plants at the center, Bhaktapur, that go with this unit. Book a class visit at science-playground.com/booking.

- **The Balancing Bird** (exhibit): The bird balances on its beak because its weighted wings put its center of gravity below the support: tilt it, and its weight turns it back.
- **Uphill Cone - Defying Gravity?** (exhibit): The cone seems to roll uphill, but watch its middle: as the rails spread apart, its center of gravity sinks lower the whole way.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	6	15	8	**29**

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