Unit 7 · Gravitation

ANSWER KEY · FOR TEACHERS

Full marks 32

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. G
2. field strength
3. potential
4. escape velocity
5. orbital velocity
6. geostationary
7. weightlessness
8. center of gravity

Each carries 0.5 mark. **K · 4**

2 Label the diagram

1. d
2. R − d
3. h
4. R

Each carries 0.5 mark. **K · 2**

3 Multiple choice

i. **b.** N m^2/kg^2 **K · 1**

From $F = Gm_1m_2/r^2$, $G = Fr^2/(m_1m_2)$, so its unit is N m^2/kg^2.

ii. **c.** g/2 **A · 1**

$g = GM/R^2$: $2M \div (2R)^2 = 2/4 = ½$ of the Earth's.

iii. **c.** the mass and radius of the Earth **U · 1**

$v_e = \sqrt{2GM/R}$: the body's own mass cancels, so a pebble and a rocket need the same speed.

iv. **c.** she and the station are falling freely round the Earth together **U · 1**

Gravity still pulls on her there: at about 400 km up, $g' = gR^2/(R + h)^2$ is nearly as large as on the ground. She and the station fall together, so nothing pushes on her: she feels weightless.

v. **c.** negative, equal to −(its kinetic energy) **U · 1**

$K = GMm/2r$ and $U = -GMm/r$, so $E = -GMm/2r = -K$. It is negative because the satellite is bound to the Earth.

4 Short answers

a. **MARKING SCHEME** **U · 2**

- ✓ The equatorial bulge: a larger distance from the center, so a smaller g.
- ✓ The Earth's rotation: part of the pull gives the centripetal force at the equator (none at the poles).

b. **MARKING SCHEME** **U · 2**

- ✓ Some gas molecules move faster than the Moon's low escape velocity, so they escape.
- ✓ The Earth's escape velocity is much higher, so its air stays.

c. **MARKING SCHEME** **A · 3**

- ✓ $g' = gR^2/(R+h)^2$.
- ✓ R + h = 8000 km, the ratio 0.64.
- ✓ $g' = 6.27$ m/s^2.

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5 Long answer

ANSWER U · 4

Gravity gives the centripetal force: $\frac{GMm}{r^2} = \frac{mv^2}{r}$, so $mv^2 = \frac{GMm}{r}$ and $K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$.

The potential energy, zero at infinity, is $U = -\frac{GMm}{r}$.

Total energy $E = K + U = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$.

E is negative: the satellite is bound to the Earth. To set it free (to raise E to zero), at least $\frac{GMm}{2r}$ of energy must be given to it.

MARKING SCHEME

- ✓ $GMm/r^2 = mv^2/r$, so $K = GMm/2r$.
- ✓ $U = -GMm/r$.
- ✓ $E = -GMm/2r$.
- ✓ Negative: the satellite is bound; $GMm/2r$ is needed to free it.

6 Numerical problems

a. **ANSWER** A · 4

$r = 6400 + 600 = 7000$ km $= 7.0 \times 10^6$ m. $GM = gR^2 = 9.8 \times (6.4 \times 10^6)^2 = 4.01 \times 10^{14}$ m^3/s^2.

$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{4.01 \times 10^{14}}{7.0 \times 10^6}} =$ **7.57 km/s**.

$T = 2\pi r/v = 2\pi \times 7.0 \times 10^6 \div 7570 = 5810$ s $\approx$ **97 minutes**.

MARKING SCHEME

- ✓ $r = 7.0 \times 10^6$ m and $GM = gR^2$.
- ✓ $v = \sqrt{(GM/r)}$.
- ✓ $v = 7.57$ km/s.
- ✓ $T \approx 97$ minutes.

b. **ANSWER** A · 3

Take the Earth's center as the origin and the Moon's mass as m: $X = \frac{81m \times 0 + m \times 3.84 \times 10^5}{82m} =$ **4680 km** from the Earth's center, towards the Moon.

That is less than the Earth's radius, 6400 km: **inside the Earth**, about 1700 km below the surface.

MARKING SCHEME

- ✓ $X = \Sigma mx/\Sigma m$ with the Earth at 0.
- ✓ $X = 4680$ km.
- ✓ Less than 6400 km, so inside the Earth.

7 Think

ANSWER HA · 3

No. Every orbit must have the Earth's center at its center, since gravity pulls towards it. An orbit that stays over 28° N all day would have to circle the Earth's axis with its center north of the Earth's center, which gravity can't give. Only an orbit over the equator turns with the Earth and keeps the satellite over one place, so from Kathmandu a geostationary satellite stays at one point low in the southern sky.

MARKING SCHEME

- ✓ No.
- ✓ An orbit's plane must pass through the Earth's center (gravity points there).
- ✓ Only an equatorial orbit stays over one place; seen from Kathmandu it is in the southern sky.

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8 Activity: Find the center of gravity

(not marked)

WHAT STUDENTS SHOULD FIND

The three lines meet at (or very nearly at) one point: the center of gravity, since a hanging body comes to rest with its center of gravity directly below the point of support. The card balances on a pencil tip at that point. A small triangle where the lines cross shows errors in drawing the lines.

See it at Science Playground

Exhibits and plants at the center, Bhaktapur, that go with this unit. Book a class visit at science-playground.com/booking.

- **The Balancing Bird** (exhibit): The bird balances on its beak because its weighted wings put its center of gravity below the support: tilt it, and its weight turns it back.
- **Uphill Cone - Defying Gravity?** (exhibit): The cone seems to roll uphill, but watch its middle: as the rails spread apart, its center of gravity sinks lower the whole way.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	7	11	11	3	**32**