Unit 6 · Circular Motion

ANSWER KEY · FOR TEACHERS

Full marks 32

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. angular velocity
2. radians
3. period
4. frequency
5. centripetal force
6. centrifugal force
7. banking
8. critical velocity

Each carries 0.5 mark. **K · 4**

2 Label the diagram

1. r
2. T cos θ
3. T sin θ
4. mg

Each carries 0.5 mark. **K · 2**

3 Multiple choice

i. **b.** rad/s **K · 1**

$\omega = \theta/t$, an angle in radians divided by a time in seconds.

ii. **c.** along the tangent to the circle at the point where it was let go **U · 1**

With no force towards the center, nothing turns its velocity, and its velocity was along the tangent.

iii. **b.** 31.4 rad/s **A · 1**

$f = 300 \div 60 = 5$ rev/s, so $\omega = 2\pi f = 10\pi \approx 31.4$ rad/s.

iv. **c.** 4 m/s^2 **A · 1**

$a = v^2/r = 400 \div 100 = 4$ m/s^2.

v. **a.** zero **U · 1**

At the critical velocity, $\sqrt{(gr)}$, the weight alone gives the centripetal force: $mg = mv^2/r$, so $T = 0$.

4 Short answers

a. **MARKING SCHEME** **U · 2**

- ✓ No real outward force: the body tends to keep moving straight (inertia) as the bus turns.
- ✓ The bus must push them inward (centripetal force); the outward push is a pseudo-force of the rotating frame.

b. **MARKING SCHEME** **U · 2**

- ✓ Friction (inwards) gives the centripetal force; with the normal reaction, the resultant is tilted inwards.
- ✓ She leans so that this resultant passes through her center of gravity, or she would topple outwards.

c. **MARKING SCHEME** **A · 3**

- ✓ $\mu mg = mv^2/r$ (friction provides the centripetal force).
- ✓ $v = 15.7$ m/s.
- ✓ About 56 km/h.

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5 Long answer

ANSWER U · 4

At the top, at the least speed, T = 0 and mg = mv_t^2/r, so v_t^2 = gr.
From the top to the bottom the stone falls 2r, so by conservation of energy ½mv_b^2 = ½mv_t^2 + mg(2r), giving v_b^2 = gr + 4gr = 5gr, so $v_b = \sqrt{5gr}$.
At the bottom the tension pulls up and the weight down: T − mg = mv_b^2/r = 5mg, so **T = 6mg**.

MARKING SCHEME

- ✓ At the top v_t^2 = gr (T = 0).
- ✓ Energy: ½mv_b^2 = ½mv_t^2 + 2mgr.
- ✓ v_b^2 = 5gr.
- ✓ At the bottom T − mg = mv²/r, so T = 6mg.

6 Numerical problems

a. **ANSWER** A · 4

i. T cos θ = mg, so T = 0.2 × 9.8 ÷ cos 30° = **2.26 N**.

ii. r = l sin θ = 1.0 × 0.5 = **0.5 m**. tan θ = v²/(rg), so v = $\sqrt{(rg \tan \theta)}$ = $\sqrt{(0.5 \times 9.8 \times 0.577)}$ = **1.68 m/s**.

iii. Period = $2\pi\sqrt{\frac{l\cos\theta}{g}} = 2\pi\sqrt{\frac{0.866}{9.8}}$ = **1.87 s**.

MARKING SCHEME

- ✓ T = mg/cos θ = 2.26 N.
- ✓ r = 0.5 m.
- ✓ v = 1.68 m/s.
- ✓ Period 1.87 s.

b. **ANSWER** A · 3

At the top: v = $\sqrt{(gr)}$ = $\sqrt{(9.8 \times 0.9)}$ = **2.97 m/s**.
At the bottom: v = $\sqrt{(5gr)}$ = $\sqrt{(5 \times 9.8 \times 0.9)}$ = **6.64 m/s**.

MARKING SCHEME

- ✓ $v_{top} = \sqrt{(gr)}$.
- ✓ v_{top} = 2.97 m/s.
- ✓ $v_{bottom} = \sqrt{(5gr)}$ = 6.64 m/s.

7 Think

ANSWER HA · 3

i. At the top its velocity is horizontal, along the tangent: it flies off horizontally and falls under gravity, a parabola, like a projectile fired horizontally.

ii. At the bottom its velocity is also horizontal: it moves off horizontally from the lowest point, again on a parabola, falling towards the ground.

In both cases it leaves along the tangent; after that only gravity acts.

MARKING SCHEME

- ✓ It leaves along the tangent at the point where the string breaks.
- ✓ At the top and at the bottom the tangent is horizontal.
- ✓ After that it is a projectile: a parabolic path under gravity.

8 Activity: Centripetal force with a string

(not marked)

WHAT STUDENTS SHOULD FIND

The keys' weight is the tension, which gives the stopper its centripetal force, so m v²/r stays about the same: with a longer radius the stopper must go faster (and v²/r comes out roughly the same in each row). The values scatter because the circle is not perfectly horizontal and the tube rubs on the string.

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- **Water Parabola** (exhibit): Spin the base: each bit of water needs a centripetal force, mv^2/r, and the greater pressure of deeper water farther out supplies it.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	7	10	12	3	**32**

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