Unit 5 · Work, Energy and Power

HARDER PRACTICE QUESTIONS

Full marks 33

Write your answers on separate paper. Show your working. Take g = 9.8 m/s^2. The settings are real kinds of things, but their numbers are made up unless a question says otherwise.

1 A bungee jump: a cord and gravity together [8]

A bungee jump is planned from a bridge over a deep gorge. The cord is 20 m long when slack and, once taut, pulls like a spring with $k = 80$ N/m (it obeys Hooke's law and its own mass can be ignored). A jumper of mass 60 kg steps off from rest; treat the jumper as a point at the end of the cord, and ignore air resistance.

a. How far below the bridge is the jumper at the lowest point?

b. Nirjala wrote $mg \times 20 = \frac{1}{2}\,kx^2$ and got a stretch of 17.1 m. What did she leave out?

c. Where is the jumper moving fastest, and at what speed?

d. Find the cord's greatest pull on the jumper, as a number of times the jumper's weight.

e. The owner wants a longer, more thrilling fall, with the lowest point 50 m below the bridge, using a softer cord of the same 20 m length. Find the k the cord must have. Does the deeper fall mean a harder pull at the bottom? Explain the result.

2 A bus on a hill road [5]

A loaded bus of mass 12 000 kg runs on a hill road that rises 1 m for every 12 m along it. Its engine can give at most 150 kW, and the resistance to its motion (friction and air) is a steady 2000 N.

a. Going down the hill, the driver keeps the bus at a steady 10 m/s with the brakes alone. At what rate do the brakes turn energy into heat? Why do drivers on long descents keep the bus in a low gear, so that the engine helps to hold it back, instead of using only the brakes?

b. Going up the hill at full power, what is the bus's greatest speed?

c. What is its acceleration at full power when it is going up at 6.0 m/s?

d. By $F = P/v$, at full power and 0.5 m/s the engine would push the bus with 300 000 N. Why can't that happen?

3 Two sledges, one stopping place [5]

Students let a sledge go from rest at the same height, 6.0 m above a long flat field of snow, down two smooth-cornered slopes, one at 20° and one at 30°. They find that it reaches the foot of the steeper slope faster (9.9 m/s against 9.2 m/s), yet both times it comes to rest about 60 m, measured horizontally, from where it started. Take the coefficient of kinetic friction μ to be the same on the slopes and the flat, and ignore air resistance.

a. Show that a sledge let go from height h stops a horizontal distance h/μ from its start, whatever the angle of the slope, and find μ from the measurements.

b. Pasang says: "The sledge was faster at the foot of the 30° slope, so the measurement for that slope must be wrong: it should have gone farther." Explain why both measurements can be right.

c. Where does the h/μ rule fail if the slope is only 5°?

4 Slowing neutrons down [5]

In a nuclear reactor, fast neutrons must be slowed down by letting them collide with the nuclei of a material, the moderator. Treat each collision as a head-on elastic collision of a neutron (mass 1 unit) with a nucleus at rest (mass m units).

a. Starting from the unit's result $v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1$ for a body hitting one at rest, show that the fraction of its kinetic energy that the moving body hands to the one at rest is

$$f = \frac{4m_1 m_2}{(m_1 + m_2)^2}$$

b. Find f for a neutron hitting a hydrogen nucleus (m = 1), a carbon nucleus (m = 12) and a lead nucleus (m = 207). Which makes the best moderator? Someone says "the heavier the nucleus, the harder the neutron hits it, so the more energy it takes": what is wrong with that?

c. A neutron must lose all but 1 part in 8×10^7 of its kinetic energy. If every collision with carbon were head-on, how many collisions would that take?

5 A ball, a cart and a spring [5]

A cart of mass 0.90 kg rests on a smooth level track, with a light spring ($k = 250$ N/m) between it and a wall beyond it. A ball of mass 0.10 kg moving at 10 m/s along the track hits the cart head-on. In one test the ball is a lump of clay that sticks to the cart; in another it is a hard steel ball that bounces straight back, in an elastic collision. The collisions are over before the spring has been squeezed noticeably.

a. Before calculating: in which test is the spring squeezed more? Give a reason.

b. Find the greatest squeeze of the spring in the clay test.

c. For the clay test, Anjila wrote: "$\frac{1}{2} \times 0.10 \times 10^2 = \frac{1}{2} \times 250x^2$, so x = 0.20 m." Explain her mistake.

d. Find the greatest squeeze in the steel test, and check your answer to a.

6 Think: can friction speed something up? [5]

A pickup starts from rest and speeds up steadily at 2.0 m/s^2 for 5.0 s on a level road. A 30 kg gas cylinder stands in its open back and doesn't slide.

a. Which force speeds the cylinder up? How much work does that force do on the cylinder in the 5.0 s?

b. Samikshya says: "Friction always opposes motion, so it always does negative work; and static friction does no work at all, because nothing slides." Show where each part of her statement goes wrong here, and say what each part gets right.

c. Where does the energy the cylinder gains come from?