Unit 5 · Work, Energy and Power

HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 33

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

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1 A bungee jump: a cord and gravity together

ANSWER HA · 8

Tests: energy with a spring and gravity at once (the stretch is also a drop); the greatest speed is where the forces balance (Unit 4), not where the cord goes taut; a design with its trade-off.

a. Let the stretch at the lowest point be x. The jumper starts and ends at rest, so by the work–energy theorem the total work on the jumper is zero. Gravity does $mg(20 + x)$ over the whole fall; the cord does $-\frac{1}{2}kx^2$ (minus the area under its force–stretch graph):

$$60 \times 9.8(20 + x) = \frac{1}{2} \times 80x^2 \quad \Rightarrow \quad 40x^2 - 588x - 11\,760 = 0$$

$$x = \frac{588 + \sqrt{588^2 + 4 \times 40 \times 11\,760}}{80} = \frac{588 + 1492.4}{80} = 26.0 \text{ m}$$

(the other root is negative). The lowest point is 20 + 26.0 = **46.0 m** below the bridge.

b. She used only the 20 m of free fall. The jumper keeps falling while the cord stretches, so gravity keeps doing work, mgx, all the way down: the drop is $20 + x$, not 20. Her stretch (17.1 m) is too short by about 9 m, a dangerous mistake for whoever measures the gorge.

c. While the cord pulls less than the weight, the net force is still downward and the jumper still speeds up. The speed is greatest where the pull equals the weight: $kx = mg$, so $x = 588/80 = 7.35$ m, that is **27.4 m below the bridge** (not at 20 m, where the cord goes taut).

$$\frac{1}{2}mv^2 = mg(20 + 7.35) - \frac{1}{2}k(7.35)^2$$

$$\frac{1}{2}mv^2 = 60 \times 9.8 \times 27.35 - 40 \times 54.0 = 16\,082 - 2161 = 13\,921 \text{ J}$$

$$v = \sqrt{2 \times 13\,921/60} = 21.5 \text{ m/s}$$

(At 20 m, where the cord goes taut, it is $\sqrt{2 \times 9.8 \times 20} = 19.8$ m/s.)

d. $kx = 80 \times 26.0 = 2080$ N, and the weight is 588 N: 2080/588 = **3.5 times** the weight.

e. Now $x = 50 - 20 = 30$ m, and $mg(20 + 30) = \frac{1}{2}k(30)^2$:

$$k = \frac{2 \times 588 \times 50}{900} = 65.3 \text{ N/m}$$

Greatest pull = 65.3 × 30 = 1960 N, 1960/588 = **3.3 times** the weight: **less** than with the stiffer cord, though the fall is deeper. The cord must take away all the energy of the fall, $mg \times$ (drop). That energy is the area under the force–stretch graph, a triangle $\frac{1}{2}F_{max}x$. A soft cord stretches much farther, so it can take the energy away with a smaller greatest force. (The tempting answer: "a deeper fall, more energy, so a harder pull".)

MARKING SCHEME

- ✓ The energy equation with the drop $20 + x$: $mg(20 + x) = \frac{1}{2}kx^2$.
- ✓ x = 26.0 m, so the lowest point is 46.0 m below the bridge.
- ✓ Nirjala left out gravity's work mgx during the stretch.
- ✓ The greatest speed is where $kx = mg$: 7.35 m of stretch, 27.4 m down.
- ✓ v = 21.5 m/s.
- ✓ Greatest pull 2080 N, 3.5 times the weight.
- ✓ k = 65.3 N/m.
- ✓ Its pull is only 3.3 times the weight: the same kind of energy is taken away over a longer stretch.

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2 A bus on a hill road

ANSWER A · 5

Tests: P = Fv at full power means the driving force falls as the speed rises; power taken away continuously by brakes on a descent; where F = P/v fails.

The pull of gravity along the road is $mg\sin\theta = 12\,000 \times 9.8 \times \frac{1}{12} = 9800$ N.

a. Going down at a steady speed the net force is zero: brakes = 9800 − 2000 = 7800 N, opposing the motion. The power they take away is $Fv = 7800 \times 10 =$ **78 kW**, every second of the descent: the potential energy the bus loses (less what resistance takes) goes into the brakes as heat. On a long descent they get hotter and hotter, and hot brakes grip less and can fail. In a low gear the engine's own resistance takes part of the power, so the brakes stay cool enough to work when needed.

b. At the greatest speed the bus no longer speeds up, so the driving force just balances 9800 + 2000 = 11 800 N: $v = P/F = 150\,000/11\,800 =$ **12.7 m/s** (about 46 km/h).

c. Driving force = 150 000/6.0 = 25 000 N, so $a = \frac{25\,000 - 11\,800}{12\,000} =$ **1.1 m/s²**. (The tempting idea: full power gives a steady acceleration. It doesn't: the force, and so the acceleration, falls as the speed rises, to zero at 12.7 m/s.)

d. The push on the bus comes from the road's friction on the driving wheels (Unit 4), and it can't be more than the limiting friction, μ times the normal reaction. The normal reaction on all the wheels together is at most the bus's weight, $12\,000 \times 9.8 = 118\,000$ N, and μ for tyres on a road is about 1 or less, so the push can never reach 300 000 N; past the limit the wheels spin. (The engine also can't give full power while turning slowly, which is what the gears are for.) So at low speed the force is set by grip and gearing, not by P/v.

MARKING SCHEME

- ✓ Down: brake force 7800 N, so 78 kW of heat.
- ✓ The heat builds up in the brakes; the engine in a low gear shares the work, so they don't overheat.
- ✓ Up: greatest speed when $P/v = 11\,800$ N: 12.7 m/s.
- ✓ At 6.0 m/s: force 25 000 N, a = 1.1 m/s^2.
- ✓ The force can't exceed the limiting friction at the wheels (or the engine can't give full power at low speed).

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3 Two sledges, one stopping place

ANSWER HA · 5

Tests: friction's work depends on the path in a way that can be worked out; two quantities that move apart (the speed at the foot and the total reach); a case where the rule fails.

a. The step to find is friction's work on the slope. On a slope of angle θ the normal reaction is $mg\cos\theta$, so the friction is $\mu mg\cos\theta$. Over a length L of slope the work against it is $\mu mg\cos\theta \times L = \mu mg(L\cos\theta)$, and $L\cos\theta$ is the horizontal distance covered. On the flat, $\cos\theta = 1$ and it is μmg times the distance too. The sledge starts and ends at rest, so all the potential energy it loses goes into work against friction: $mgh = \mu mg\,D$, where D is the whole horizontal distance, slope and flat together. So $D = h/\mu$, with no θ in it. From the measurements, $\mu = h/D = 6.0/60 =$ **0.10**.

b. On the steeper slope the sledge covers less horizontal distance on the slope (10.4 m against 16.5 m), so friction takes less energy there, and it arrives faster. But then it has farther to go on the flat, where friction takes the extra energy at the same rate per meter. Friction's total is μmg times the total horizontal distance either way, so both stop at the same place: 49.6 m of flat after the 30° slope, 43.5 m after the 20° one. (The tempting answer is Pasang's: "faster at the foot, so farther".)

c. At 5°, $\tan 5° = 0.087$, less than $\mu = 0.10$: the pull down the slope, $mg\sin\theta$, is less than friction, $\mu mg\cos\theta$, so the sledge doesn't slide at all (and a pushed one slows and stops on the slope). The rule assumed it slides all the way down; it works only for slopes steeper than about 5.7° ($\tan\theta > \mu$).

MARKING SCHEME

- ✓ Friction $\mu mg\cos\theta$ along a length L does $\mu mg \times$ horizontal distance.
- ✓ $mgh = \mu mgD$, so $D = h/\mu$, independent of θ.
- ✓ $\mu = 6.0/60 = 0.10$.
- ✓ The steep slope loses less on the slope, but has a longer stretch of flat; the total friction work, and so the stopping place, is the same.
- ✓ At 5°, $\tan\theta < \mu$: the sledge doesn't slide, so the rule doesn't apply.

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4 Slowing neutrons down

ANSWER

Tests: the energy handed over in an elastic collision depends on the mass ratio, greatest for equal masses; the unit's three special cases as one formula; combining a fraction kept per collision over many collisions.

a. The moving body keeps the fraction $\left(\frac{v_1}{u_1}\right)^2 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$ of its kinetic energy ($K \propto v^2$). The rest goes to the other body, since kinetic energy is conserved:

$$f = 1 - \frac{(m_1 - m_2)^2}{(m_1 + m_2)^2} = \frac{(m_1 + m_2)^2 - (m_1 - m_2)^2}{(m_1 + m_2)^2} = \frac{4m_1 m_2}{(m_1 + m_2)^2}$$

b. With $m_1 = 1$: hydrogen $f = 4/4 = \mathbf{1}$ (all of it); carbon $f = 48/169 = \mathbf{0.28}$; lead $f = 828/43\,264 = \mathbf{0.019}$ (2%). **Hydrogen** is best: equal masses exchange velocities, so one head-on hit stops the neutron (the unit's case a). A heavy nucleus hardly moves: the neutron bounces off it at almost its full speed, like a ball off a wall (case b), so it keeps nearly all its energy. A hard hit changes the neutron's **velocity** a lot (it reverses), not its **speed**; only a nucleus that moves off fast can carry energy away.

c. Each head-on collision with carbon leaves the fraction $1 - 0.284 = 0.716$. After n collisions $0.716^n = 1/(8 \times 10^7)$:

$$n = \frac{\log(8 \times 10^7)}{\log(1/0.716)} = \frac{7.90}{0.145} = 54.5$$

so **55 collisions**. (Most real collisions are glancing and hand over less, so a real neutron needs more. 8×10^7 is about the ratio of a neutron's energy when it is born in a reactor to its energy once slowed to the temperature of its surroundings.)

MARKING SCHEME

- ✓ Fraction kept $((m_1 - m_2)/(m_1 + m_2))^2$, from $K \propto v^2$.
- ✓ $f = 1 -$ that $= 4m_1 m_2/(m_1 + m_2)^2$.
- ✓ f = 1, 0.28 and 0.019 for H, C and Pb.
- ✓ Hydrogen is best; a heavy nucleus barely moves, so the neutron rebounds keeping its speed and energy.
- ✓ $0.716^C = 1/(8 \times 10^7)$, so 55 collisions.

5 A ball, a cart and a spring

ANSWER

Tests: two stages, two laws (momentum through the collision, mechanical energy after it); a bouncing ball gives a target more momentum than one that sticks, an unexpected result.

a. In the **steel** test. The cart gets the momentum the ball loses. The clay ball stops, relative to the cart, so it loses only part of its momentum; the steel ball reverses, so it loses more than all of it (from +1.0 to a backward momentum). More momentum given to the cart means a faster cart and a bigger squeeze. (The tempting answer: the sticking ball, because "it puts all its energy into the cart".)

b. Collision, momentum conserved: $0.10 \times 10 = (0.10 + 0.90)V$, so $V = 1.0$ m/s. Then, with only the spring doing work, mechanical energy is conserved: $\frac{1}{2} \times 1.00 \times 1.0^2 = \frac{1}{2} \times 250x^2$, so $x = \sqrt{0.0040} = \mathbf{0.063\ m}$ (6.3 cm).

c. She assumed the ball's kinetic energy all reached the spring. But the clay collision is perfectly inelastic: of the ball's 5.0 J only 0.50 J is left as kinetic energy after it (90% becomes heat and the squashing of the clay). Kinetic energy isn't conserved through an inelastic collision; only momentum is, so the collision must be done with momentum first.

d. Elastic, cart at rest (the unit's equations 5.4 and 5.5): ball $v_1 = \frac{0.10 - 0.90}{1.00} \times 10 = -8.0$ m/s, cart $v_2 = \frac{2 \times 0.10}{1.00} \times 10 = 2.0$ m/s. $\frac{1}{2} \times 0.90 \times 2.0^2 = 1.8$ J $= \frac{1}{2} \times 250x^2$, so $x = \sqrt{0.0144} = \mathbf{0.12\ m}$, nearly twice the clay test's, as predicted. (Check: momentum $0.10 \times (-8.0) + 0.90 \times 2.0 = 1.0$, kinetic energy $3.2 + 1.8 = 5.0$ J.)

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MARKING SCHEME HA · 5

- ✓ Steel: the bouncing ball loses more momentum, so it gives the cart more.
- ✓ Clay: V = 1.0 m/s from momentum.
- ✓ Clay: x = 0.063 m from ½(m + M)V^2 = ½kx^2.
- ✓ Her mistake: kinetic energy is lost in the inelastic collision (only 0.50 J of 5.0 J remains).
- ✓ Steel: cart 2.0 m/s, x = 0.12 m, more than the clay test.

6 Think: can friction speed something up?

ANSWER HA · 5

Tests: friction can do positive work; "static friction does no work" is true only for a body that doesn't move; friction opposes slipping between surfaces, not motion.

a. The only horizontal force on the cylinder is **static friction** from the floor of the pickup, pointing forward: $f = ma = 30 \times 2.0 = 60$ N. In 5.0 s the cylinder moves with the pickup $\frac{1}{2} \times 2.0 \times 5.0^2 = 25$ m, in the direction of the friction, so the friction does $60 \times 25 =$ **+1500 J** on it. Check by the work–energy theorem: its speed is $2.0 \times 5.0 = 10$ m/s, and $\frac{1}{2} \times 30 \times 10^2 = 1500$ J.

b. Friction opposes **slipping** of one surface over another, not motion. Here the floor moves forward under the cylinder and would slide out from under it; friction stops that by pulling the cylinder forward, along its motion, so its work is positive. What she gets right: for a body sliding over a fixed surface (a box pushed along a floor), friction is against the motion and its work is negative. Static friction does no work on a body that stays **at rest** (no displacement), as the unit says. The cylinder isn't at rest: it doesn't slide relative to the pickup, but it moves 25 m along the road. What she gets right: because the surfaces don't slip, no heat is made between them.

c. From the pickup's engine, ultimately the fuel: the engine drives the pickup, and through the floor's friction some of that energy goes into the cylinder. (The cylinder pulls back on the floor with 60 N, so the friction on the pickup does −1500 J: static friction passes the energy on without losing any.)

MARKING SCHEME

- ✓ Static friction from the floor, forward, 60 N.
- ✓ Work = 60 × 25 = +1500 J (= ½mv^2 with v = 10 m/s).
- ✓ Friction opposes slipping, not motion: here it acts along the motion, so positive work.
- ✓ Static friction does no work only on a body at rest; this cylinder moves 25 m.
- ✓ The energy comes from the pickup's engine (fuel), passed on through the friction.

See it at Science Playground

Exhibits and plants at the center, Bhaktapur, that go with this unit. Book a class visit at science-playground.com/booking.

- **Coupled Pendulums** (exhibit): Start just one pendulum: its swing dies away as the other's grows, then returns, the energy passing between them while their total stays nearly constant.
- **Newton's Cradle** (exhibit): Nearly elastic collisions: almost all the kinetic energy passes along the row, and the little lost each time, to sound and air, slowly stops the balls.
- **Pulley Systems** (exhibit): Lift the same load with each pulley arrangement: halve the force and you pull twice as much rope, so the work done, Fd, stays about the same.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	0	5	28	**33**

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