HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 30

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Spot the error

ANSWER U · 3

He took cos 120° as +0.5. It is **−0.5**, since 120° is more than 90°.

$R = \sqrt{100 + 100 - 100} = \sqrt{100} =$ **10 N**: two equal vectors at 120° have a resultant equal to either of them.

(A check: at 120° the forces pull partly against each other, so R must be less than at 90°, $\sqrt{200} = 14.1$ N, not more.)

MARKING SCHEME

- ✓ The mistake: cos 120° = −0.5, not +0.5.
- ✓ $R^2 = 100 + 100 - 100 = 100$.
- ✓ R = 10 N.

2 Pulling out a jeep

ANSWER A · 4

i. Along the road each rope gives 2000 cos 30° = 1732 N, so the resultant is 2 × 1732 = **3464 N** along the road.

ii. Across the road the components, 2000 sin 30° = 1000 N each, are equal and opposite, so they cancel.

iii. 2 × 2000 cos 60° = **2000 N**, only as much as one rope pulling straight: the wider the angle, the more of each pull is wasted pulling sideways against the other.

MARKING SCHEME

- ✓ Each rope's component along the road: 2000 cos 30° = 1732 N.
- ✓ Resultant 3464 N along the road.
- ✓ The sideways components (1000 N each) cancel.
- ✓ At 60°: 2000 N.

3 Show that

ANSWER A · 3

Square both sides, using $|\vec{X}|^2 = \vec{X}\cdot\vec{X}$:

$(\vec{A} + \vec{B})\cdot(\vec{A} + \vec{B}) = (\vec{A} - \vec{B})\cdot(\vec{A} - \vec{B})$

$A^2 + 2\vec{A}\cdot\vec{B} + B^2 = A^2 - 2\vec{A}\cdot\vec{B} + B^2$, so $4\vec{A}\cdot\vec{B} = 0$ and $\vec{A}\cdot\vec{B} = 0$.

For non-zero vectors, cos $\theta = 0$: $\theta = 90°$. (Or by the formula:

$P^2 + Q^2 + 2PQ\cos\theta = P^2 + Q^2 - 2PQ\cos\theta$.)

MARKING SCHEME

- ✓ Both sides squared (as dot products, or with the resultant formula).
- ✓ $\vec{A}\cdot\vec{B} = 0$ (4AB cos $\theta = 0$).
- ✓ So $\theta = 90°$.

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4 Find the unknown

ANSWER A · 4

i. $\vec{A}\cdot\vec{B} = 8 - 2\lambda - 2 = 0$, so $\lambda = \mathbf{3}$.

ii. $\vec{A}\times\vec{B} = (3\times(-2) - 1\times(-2))\hat{i} + (1\times 4 - 2\times(-2))\hat{j} + (2\times(-2) - 3\times 4)\hat{k} = \mathbf{-4\hat{i} + 8\hat{j} - 16\hat{k}}$; magnitude $\sqrt{16 + 64 + 256} = \sqrt{336} \approx \mathbf{18.3}$.

iii. $|\vec{A}\times\vec{B}| = AB\sin\theta$, and $\theta = 90°$, so $\sin\theta = 1$. Check: $|\vec{A}| = \sqrt{14}$, $|\vec{B}| = \sqrt{24}$, and $\sqrt{14}\times\sqrt{24} = \sqrt{336}$.

MARKING SCHEME

- ✓ $\lambda = 3$ from the zero dot product.
- ✓ $\vec{A}\times\vec{B} = -4\hat{i} + 8\hat{j} - 16\hat{k}$.
- ✓ Magnitude $\sqrt{336} \approx 18.3$.
- ✓ $\sin 90° = 1$, so the magnitude is AB ($\sqrt{14}\times\sqrt{24} = \sqrt{336}$).

5 Read the data

ANSWER HA · 5

i. At 0°, R = P + Q = 10.0 N; at 180°, R = P − Q = 2.0 N. So **P = 6 N, Q = 4 N**.

ii. $R = \sqrt{36 + 16 + 48\cos\theta}$: at 60°, $\sqrt{76} = 8.7$ N; at 90°, $\sqrt{52} = 7.2$ N; at 120°, $\sqrt{28} = \mathbf{5.3\ N}$. The reading at **120°, 6.3 N**, is wrong (it is 1 N too big); the others agree.

iii. R falls steadily as θ grows, from the sum (10 N) at 0° to the difference (2 N) at 180°, and is $\sqrt{(P^2 + Q^2)}$ at 90°.

MARKING SCHEME

- ✓ P + Q = 10 and P − Q = 2.
- ✓ P = 6 N, Q = 4 N.
- ✓ R worked out at 60°, 90° and 120° (8.7, 7.2, 5.3 N).
- ✓ The reading at 120° is wrong.
- ✓ R falls from P + Q to P − Q as θ goes from 0° to 180°.

6 Least and most

ANSWER A · 3

i. From 12 − 5 = **7 N** (opposite) to 12 + 5 = **17 N** (together).

ii. $13^2 = 169 = 5^2 + 12^2$, so $2\times 5\times 12\cos\theta = 0$: $\theta = \mathbf{90}°$. With the 12 N force: $\tan\alpha = 5/12$, $\alpha = \mathbf{22.6}°$.

MARKING SCHEME

- ✓ Between 7 N and 17 N.
- ✓ $\theta = 90°$ ($169 = 25 + 144$, so $\cos\theta = 0$).
- ✓ $\alpha = 22.6°$ from the 12 N force.

7 Correct the misconceptions

ANSWER U · 3

1. Speed is a **scalar**: it is only the size of the velocity; velocity is the vector.
2. A quantity with size and direction is a vector only if it also adds by the triangle law: electric current has a direction but adds as a scalar.
3. The resultant can be smaller than either vector: 5 N and 3 N in opposite directions give 2 N; two equal vectors at 120° give one of the same size.

MARKING SCHEME

- ✓ Speed is a scalar (velocity is the vector).
- ✓ It must also add by the triangle law (current is a scalar with a direction).
- ✓ The resultant can be smaller (opposite vectors, or wide angles).

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8 Think

ANSWER HA · 3

No. $|\vec{A}\times\vec{B}| = AB\sin\theta$ is largest at $\theta = 90°$, where $\vec{A}\cdot\vec{B} = AB\cos 90° = 0$, not negative. A negative dot product needs θ between 90° and 180°, where sin θ is less than 1.
$AB\cos\theta = -AB\sin\theta$ means $\tan\theta = -1$, so $\theta = \mathbf{135}°$.

MARKING SCHEME

- ✓ No: the cross product is largest at 90°, where the dot product is zero.
- ✓ A negative dot product needs $\theta > 90°$, where $\sin\theta < 1$.
- ✓ $\theta = 135°$ ($\tan\theta = -1$).

9 Make your own

ANSWER HA · 2

R = P needs $P^2 + Q^2 + 2PQ\cos\theta = P^2$, so $\cos\theta = -\frac{Q}{2P}$. For example P = 6 N, Q = 5 N: $\cos\theta = -5/12$, θ = 114.6°. Check: $R^2 = 36 + 25 + 60 \times (-5/12) = 36$, R = 6 N. (Any P, Q with Q < 2P works.)

MARKING SCHEME

- ✓ A choice with $\cos\theta = -Q/(2P)$ (the condition, or a correct trial).
- ✓ Shown to work by the resultant formula.

See it at Science Playground

Exhibits and plants at the center, Bhaktapur, that go with this unit. Book a class visit at science-playground.com/booking.

- **The Wind Sock** (exhibit): The wind sock shows which way the wind blows and, by how far it lifts, roughly how fast: the wind's velocity is a vector.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	6	14	10	**30**

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