Unit 2 · Vectors

ANSWER KEY · FOR TEACHERS

Full marks 32

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. unit vector → **f.** has a magnitude of 1 and shows a direction only
2. null vector → **a.** has zero magnitude
3. negative of a vector → **e.** has the same magnitude and the opposite direction
4. collinear vectors → **b.** act along the same line or parallel lines
5. coplanar vectors → **c.** lie in the same plane

K · 2.5

Each carries 0.5 mark.

2 True or false

i. ✓ **True**

ii. ✗ **False**: Their resultant can only be between $5 - 3 = 2$ N and $5 + 3 = 8$ N.

iii. ✓ **True**

iv. ✗ **False**: $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$: swapping the order reverses the direction.

v. ✗ **False**: It is a scalar, $AB\cos\theta$.

vi. ✓ **True**

U · 3

Each carries 0.5 mark.

3 Label the diagram

1. θ
2. A sin θ
3. Ax = A cos θ

K · 1.5

Each carries 0.5 mark.

4 Multiple choice

i. **a.** work

Work = $\vec{F} \cdot \vec{s}$, a dot product, has no direction. The other three are vectors.

K · 1

ii. **b.** 10 N

$R = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$ N ($\theta = 90°$, so $\cos\theta = 0$).

U · 1

iii. **b.** 5

$\vec{A} \cdot \vec{B} = 2 \times 4 + 3 \times (-1) = 8 - 3 = 5$, a number.

A · 1

iv. **a.** $\hat{j}$

In the cycle $\hat{i} \to \hat{j} \to \hat{k} \to \hat{i}$, a product taken in the cycle's order gives the third vector: $\hat{k} \times \hat{i} = \hat{j}$.

A · 1

v. **c.** $\sqrt{3}$

$\frac{AB\sin 60°}{AB\cos 60°} = \tan 60° = \sqrt{3}$.

U · 1

5 Short answers

a. MARKING SCHEME

✓ No.

✓ Three vectors adding to zero form a closed triangle (triangle law), which lies in one plane.

U · 2

b. MARKING SCHEME

✓ The force along the floor is the horizontal component, F cos θ.

✓ Small θ gives cos θ near 1 (most of the pull used); a steep rope gives a small horizontal component.

U · 2

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c. MARKING SCHEME A · 2

✓ $\vec{A}\cdot\vec{B} = 2 - 3 + 1 = 0$.

✓ A zero dot product of non-zero vectors means $\cos\theta = 0$, $\theta = 90°$.

6 Long answer

ANSWER U · 4

$\cos\theta$ is greatest, 1, at $\theta = 0°$: $R = \sqrt{(P+Q)^2} = P + Q$, the greatest, when the vectors are parallel.
$\cos\theta$ is least, −1, at $\theta = 180°$: $R = \sqrt{(P-Q)^2} = |P - Q|$, the least, when they are opposite.
For P = Q: $R^2 = 2P^2 + 2P^2\cos\theta = 3P^2$, so $\cos\theta = ½$ and $\theta = \mathbf{60}°$.

MARKING SCHEME

✓ Greatest at $\theta = 0°$ ($\cos\theta = 1$): R = P + Q.

✓ Least at $\theta = 180°$ ($\cos\theta = -1$): R = |P − Q|.

✓ For equal vectors: $2P^2 + 2P^2\cos\theta = 3P^2$.

✓ $\cos\theta = ½$, $\theta = 60°$.

7 Numerical problems

a. ANSWER A · 4

Take east as +x and north as +y.
x = 400 + 300 cos 60° = 400 + 150 = 550 m; y = 0 + 300 sin 60° = 259.8 m.
$d = \sqrt{550^2 + 259.8^2} = \mathbf{608\ m}$.
tan θ = 259.8 / 550 = 0.472, $\theta = \mathbf{25.3}°$ **north of east**.

MARKING SCHEME

✓ x-components added: 550 m.

✓ y-components added: 259.8 m.

✓ d = 608 m.

✓ 25.3° north of east.

b. ANSWER A · 3

$W = 3 \times 5 + 4 \times 0 = \mathbf{15\ J}$.

F = $\sqrt{(3^2 + 4^2)}$ = 5 N and s = 5 m, so $\cos\theta = \dfrac{15}{5 \times 5} = 0.6$, $\theta = \mathbf{53.1}°$.

MARKING SCHEME

✓ W = 15 J.

✓ F = 5 N, s = 5 m and $\cos\theta = 0.6$.

✓ $\theta = 53.1°$.

8 Think

ANSWER HA · 3

No. For the ring to stay at rest the three forces must add to zero, so drawn head to tail they must close a triangle. Then the resultant of any two must be equal and opposite to the third. But the resultant of 5 N and 6 N can be at most 5 + 6 = 11 N, less than 12 N, so no angle makes them balance it.

MARKING SCHEME

✓ No.

✓ At rest needs a zero resultant: the three forces must form a closed triangle (two balance the third).

✓ The largest resultant of 5 N and 6 N is 11 N, less than 12 N.

9 Activity: Three pulls on a ring

(not marked)

WHAT STUDENTS SHOULD FIND

Both sums of components come out close to zero, because a ring at rest has no resultant force on it: the three pulls form a closed triangle. They differ a little because of reading errors in the balances and the angles, and the balances' own weight and friction.

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- **The Wind Sock** (exhibit): The wind sock shows which way the wind blows and, by how far it lifts, roughly how fast: the wind's velocity is a vector.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	5	13	11	3	**32**

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