Unit 1 · Physical Quantities

HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 33

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Spot the error

ANSWER U · 3

He changed meters into centimeters but forgot that the unit is a **cubic** meter: 1 $m^3 = (100\ cm)^3 = 10^6$ cm^3, not 100 cm^3.

1.2 $kg/m^3 = 1.2 \times 1000\ g \div 10^6\ cm^3 =$ **1.2×10^{-3} g/cm³**.

(A check: 12 g/cm^3 would make air heavier than iron.)

MARKING SCHEME

- ✓ The mistake: 1 m^3 is 10^6 cm^3, not 100 cm^3 (the power 3 of the length was left out).
- ✓ $1.2 \times 10^3 \div 10^6$ worked out.
- ✓ 1.2×10^{-3} g/cm^3.

2 Find the law

ANSWER A · 4

i. Let $T = k\,r^a\,M^b\,G^c$. Then $[T] = [L]^a[M]^b[M^{-1}L^3T^{-2}]^c = [M^{b-c}L^{a+3c}T^{-2c}]$.

T: $-2c = 1$, so $c = -½$. M: $b - c = 0$, so $b = -½$. L: $a + 3c = 0$, so $a = 3/2$.

$T = k\dfrac{r^{3/2}}{\sqrt{GM}}$, so $T^2 \propto r^3$ (this is Kepler's third law; the pure number $k = 2\pi$ comes from the full theory).

ii. $T \propto r^{3/2}$, so T is $4^{3/2} =$ **8 times** as long.

MARKING SCHEME

- ✓ $T = k\ r^a\ M^b\ G^c$ with the dimensions put in.
- ✓ The three equations for M, L and T.
- ✓ $a = 3/2$, $b = -½$, $c = -½$: $T \propto r^{3/2}/\sqrt{GM}$.
- ✓ $4^{3/2} = 8$ times as long.

3 Measure g with a pendulum

ANSWER A · 5

i. $T = 40.2 \div 20 = 2.01$ s. $g = \dfrac{4\pi^2 l}{T^2} = \dfrac{4\pi^2 \times 1.000}{2.01^2} =$ **9.77 m/s²**.

ii. $g = 4\pi^2\,l\,T^{-2}$, so $\dfrac{\Delta g}{g} = \dfrac{\Delta l}{l} + 2\dfrac{\Delta T}{T} = \dfrac{0.1}{100.0} + 2 \times \dfrac{0.2}{40.2} = 0.0010 + 0.0100 = 0.011$, about **1.1%**. $\Delta g = 0.011 \times 9.77 \approx 0.1$ m/s², so g = **(9.8 ± 0.1) m/s²**.

iii. The **time**: it adds 1.0% (counted twice, since T is squared), the length only 0.1%. Timing more swings, say 40 or 50, spreads the same 0.2 s over more swings and makes ΔT/T smaller.

MARKING SCHEME

- ✓ T = 2.01 s.
- ✓ g = 9.77 m/s².
- ✓ Δg/g = Δl/l + 2ΔT/T (the power 2 used).
- ✓ About 1.1%, g = (9.8 ± 0.1) m/s².
- ✓ The time adds more; time more swings.

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking

4 Estimate it

ANSWER A · 3

$70 \times 60 \times 24 \times 365 \times 75 = 2{,}759{,}400{,}000 \approx \mathbf{3 \times 10^9}$ beats (order of magnitude 10^9).
The inputs (70 beats a minute, 75 years) are rough, known to one or two figures at most, so the answer can't be more precise than they are: writing 2,759,400,000 claims a precision of one beat in a lifetime that the estimate doesn't have.

MARKING SCHEME

- ✓ $70 \times 60 \times 24 \times 365 \times 75 \approx 2.76 \times 10^9$.
- ✓ 3×10^9, order of magnitude 10^9.
- ✓ The answer can be no more precise than the rough inputs.

5 What if?

ANSWER HA · 3

i. $25.6 - 24.9 = 0.7$ cm. In a difference the absolute uncertainties add: $0.1 + 0.1 = 0.2$ cm. So **(0.7 ± 0.2) cm**, an uncertainty of $0.2 \div 0.7 \approx$ **29%**.

ii. The uncertainties add even when the values subtract, so the 0.2 cm stays while the length shrinks to 0.7 cm. Subtracting two nearly equal readings throws away most of the measurement and keeps all the error. Such an experiment should be planned so the difference is large (for example, a longer column or a bigger change).

MARKING SCHEME

- ✓ 0.7 cm, ± 0.2 cm (absolute uncertainties added).
- ✓ About 29%.
- ✓ The uncertainties add while the value becomes small; avoid subtracting nearly equal readings (make the difference large).

6 Work backwards

ANSWER A · 3

Force is $[M\ L\ T^{-2}]$, so the unit of force must equal (unit of mass) × (unit of length) ÷ (unit of time)^2.
$1\ N = 10\ kg \times 10\ m \div t^2 = 100\ kg\ m \div t^2$. Since $1\ N = 1\ kg\ m/s^2$, $t^2 = 100\ s^2$, so $\mathbf{t = 10\ s}$.

MARKING SCHEME

- ✓ The unit of force = unit of mass × unit of length ÷ (unit of time)^2, from $[M\ L\ T^{-2}]$.
- ✓ $1\ kg\ m/s^2 = 100\ kg\ m \div t^2$.
- ✓ $t = 10$ s.

7 Which stopwatch?

ANSWER U · 3

i. A's readings spread over only 0.03 s, so **A is more precise**; but their mean, 10.305 s, is 0.3 s off. B's spread over 0.7 s, but their mean, 10.025 s, is close to 10.00 s, so **B is more accurate on average**.

ii. **B's scatter.** Random errors go both ways and cancel more and more as readings are averaged. A's error is steady (about 0.3 s every time, perhaps a slow reaction or a fast watch): averaging gives the same wrong value however many readings are taken.

MARKING SCHEME

- ✓ A more precise (spread 0.03 s); B more accurate (mean 10.025 s against A's 10.305 s).
- ✓ Averaging reduces B's random scatter.
- ✓ A's steady (systematic) error is not reduced by averaging.

8 Correct the misconceptions

ANSWER

1. 0.0500 has **three** significant figures: the leading zeros don't count, but the zeros after the 5, after a decimal point, do.
2. Extra calculator digits don't make an answer more accurate (or more precise): the answer keeps only as many figures as the least precise measurement allows.

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking

3. A dimensionally correct equation can still be wrong: dimensions can't catch a wrong number (v = u + 2at is dimensionally correct but wrong). U · 3

MARKING SCHEME

- ✓ 0.0500 has three significant figures (trailing zeros after a decimal point count).
- ✓ An answer can't be more precise than the measurements; extra digits mean nothing.
- ✓ Dimensionally correct doesn't mean correct (wrong pure numbers aren't caught).

9 Think

ANSWER HA · 4

i. Let $T = c\,m^a\,k^b\,A^d$: $[T] = [M]^a[MT^{-2}]^b[L]^d = [M^{a+b}L^dT^{-2b}]$. T: $-2b = 1$, $b = -½$. M: $a + b = 0$, $a = ½$. L: $d = 0$. So $T = c\sqrt{m/k}$.

ii. $d = 0$: the period doesn't depend on the amplitude: a small bounce and a big one take the same time.

iii. With g as well there are four unknown powers, but only three equations (one each for M, L and T). One power is left free, so many relations fit the dimensions, and dimensions alone can't choose between them.

MARKING SCHEME

- ✓ The dimensions put in and matched.
- ✓ $T = c\sqrt{(m/k)}$.
- ✓ No dependence on the amplitude ($d = 0$).
- ✓ With g there are four unknowns and only three equations, so the powers can't all be found.

10 Make your own

ANSWER HA · 2

For example: $v = u + 3at$ is dimensionally correct (every term is $[L\,T^{-1}]$) but wrong: the true relation is $v = u + at$.

$v = u + at^2$ is dimensionally wrong: at^2 is [L], a length, not a velocity.

MARKING SCHEME

- ✓ A dimensionally correct but wrong equation, with every term shown to be $[L\,T^{-1}]$.
- ✓ A dimensionally wrong one, with the term whose dimensions differ.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	9	15	9	**33**

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking