Unit 24

Nuclear Physics

This unit online, with its interactives:
science-playground.com/s/11f24

One kilogram of coal, burnt, keeps a 1 kW heater going for about eight hours. One kilogram of uranium-235, split in a reactor, could keep the same heater going for over two thousand years. The difference is not in the fuel's chemistry but in where the energy is stored: in coal it sits between atoms, in uranium it sits inside the tiny nucleus at the center of each atom.

This unit opens that nucleus. It starts with the experiment that found it, a beam of alpha particles fired at gold leaf, and measures its size from how close an alpha particle can get. Then it asks what holds a nucleus together when every proton in it pushes every other one away, and answers with Einstein's $E = mc^2$: a nucleus weighs **less** than the parts it is made of, and the missing mass is the energy that binds it. One graph, the binding energy per nucleon, then explains why the Sun shines by joining light nuclei and why a power station can run by splitting heavy ones.

24.1 The discovery of the nucleus: Rutherford's alpha-scattering experiment

Ernest Rutherford won the 1908 Nobel Prize in Chemistry for his work on radioactivity, and only afterwards made his greatest discovery. Years later he described the moment his assistants told him that an alpha particle could bounce straight back off a sheet of gold:

> "It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you."

In 1911, from this experiment, he proposed that all the positive charge of an atom, and nearly all its mass, is packed into a tiny region at its center: the **nucleus**.

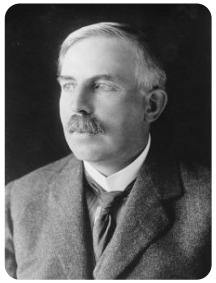


Figure 24.1 *Ernest Rutherford (1871–1937), born in New Zealand, worked in Canada and England.*
Photo: George Grantham Bain Collection (Library of Congress), Public domain

The experiment

The experiment was carried out in Rutherford's laboratory in Manchester by Hans Geiger and Ernest Marsden from 1909 to 1913. Figure 24.2 shows a simplified version of it.

- An **alpha particle** is a helium nucleus, two protons and two neutrons, with a charge of $+2e$. It comes out of a radioactive source such as radium at about 5 % of the speed of light.
- The foil was gold because gold can be beaten thinner than any other metal, down to well under a micrometer.
- Each alpha particle hitting the zinc sulphide screen made a faint speck of light, a **scintillation**. Geiger and Marsden sat in the dark and counted the flashes by eye, at each angle θ from 0° to nearly 180°, often for hours.

Observations

- **Most alpha particles went straight through** the foil, or were turned through only a degree or two.
- **A very few were turned through large angles.** About 1 alpha particle in 8000 was turned through **more than 90°**, coming back off the foil on the same side it went in.

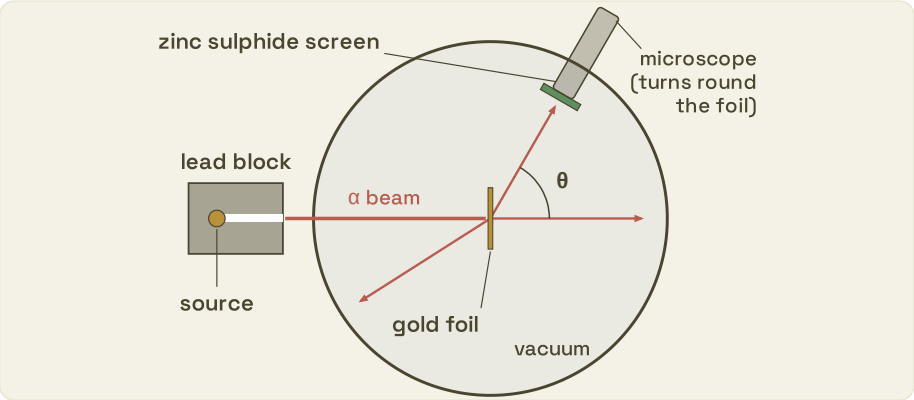


Figure 24.2 *Rutherford's alpha-scattering experiment. A narrow beam of alpha particles, all with the same kinetic energy, leaves a radioactive source in a lead block and strikes a gold foil only a few hundred atoms thick. Each alpha particle that is scattered through the angle θ makes a tiny flash on a zinc sulphide screen, seen through a microscope that can be swung round the foil. The air is pumped out of the container so that the alpha particles aren't stopped by air.*

CAREFUL

Read the right numbers

Books quote different fractions for these observations, and they don't all fit together. What Geiger and Marsden reported in 1909, from a foil of platinum, gold's neighbor in the periodic table, was that about 1 alpha particle in 8000 was turned back through more than 90°. All the other counts, at all the other angles, follow from Rutherford's formula and the energy of the beam, so there is no single number for "how many were deflected": the count falls off steeply as the angle grows.

Conclusions: the nuclear model

- **The atom is mostly empty space.** Most alpha particles passed through a few hundred layers of atoms as if nothing were there.
- **The positive charge, and almost all the mass, sits in a tiny nucleus.** To turn a fast, heavy alpha particle right round, a single atom must push on it with a huge force, and only a charge packed into a very small region can do that (the push of a point charge grows as $1/r^2$, without limit, as you get closer). One close encounter with one nucleus, not many small nudges, turns it back.
- **The nucleus is positive**, because it repels the positive alpha particles, and **much heavier than an alpha particle**, because it is not knocked out of the way itself.

Play the experiment yourself. Each path below is worked out from the push of a gold nucleus on a 7.7 MeV alpha particle.

Check 1

Why did the alpha particles that bounced back show that the atom's positive charge is concentrated in a tiny nucleus?

a because they had lost all their energy in the foil
b because they hit electrons, which are very heavy
c because only a charge packed into a tiny region can push hard enough to turn a fast alpha particle right round
d because gold atoms are larger than other atoms

Sizes of the atom and the nucleus

Rutherford estimated the size of the gold nucleus from how close an alpha particle can get to it.

Think of an alpha particle aimed **head-on** at a nucleus. As it approaches, the repulsion slows it down, turning its kinetic energy E_K into electric potential energy E_P. At the **distance of closest approach**, d, it stops for an instant, and

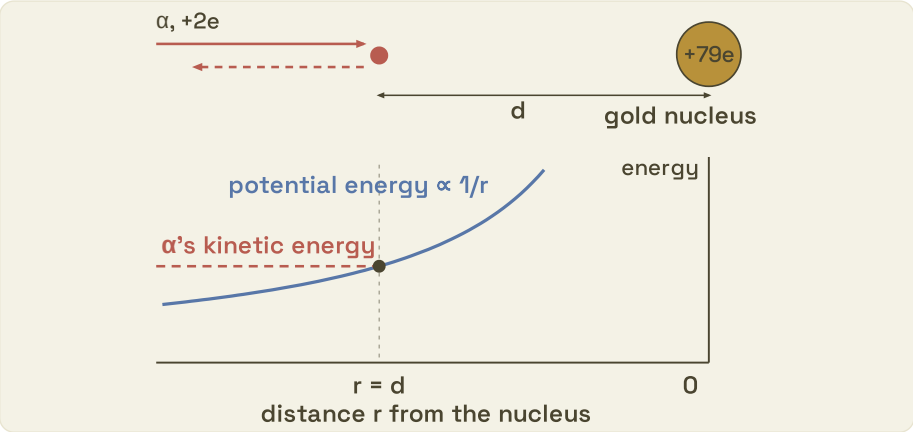


Figure 24.3 *An alpha particle fired straight at a gold nucleus slows down as the repulsion grows. At the distance of closest approach d it stops for an instant: all its kinetic energy has become electric potential energy. Then it is pushed back the way it came.*

$$E_P = E_K$$

The electric potential energy of two charges q_1 and q_2 a distance r apart is (Unit 21)

$$E_P = \frac{kq_1q_2}{r}, \qquad k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$$

The alpha particle has charge $q_1 = +2e$; a nucleus of atomic number Z has $q_2 = +Ze$ (for gold, $Z = 79$). So, at the closest approach,

$$E_K = \frac{k(2e)(Ze)}{d} \quad \Rightarrow \quad d = \frac{k(2e)(Ze)}{E_K} = \frac{2kZe^2}{E_K}$$

The alpha particle never touches the nucleus, so the nucleus's radius must be **smaller than** d. The distance of closest approach is an **upper limit** for the nuclear radius.

WORKED EXAMPLE

Closest approach of a 4.0 MeV alpha particle

Alpha particles of kinetic energy 4.0 MeV (typical for alpha particles) are fired head-on at a gold nucleus ($Z = 79$). How close do they get? (1 MeV $= 1.6 \times 10^{-13}$ J; $e = 1.6 \times 10^{-19}$ C.)

Answer

$E_K = 4.0 \times 10^6 \text{ eV} = 4.0 \times 10^6 \times 1.6 \times 10^{-19} = 6.4 \times 10^{-13}$ J.

$$d = \frac{2kZe^2}{E_K} = \frac{8.99 \times 10^9 \times (2 \times 79) \times (1.6 \times 10^{-19})^2}{6.4 \times 10^{-13}} = \mathbf{5.7 \times 10^{-14}\ m}$$

So the gold nucleus has a radius **less than about 10^{-13} m**, on the order of 10^{-14} m. This is the size Rutherford concluded.

Later, sharper experiments (fast electrons fired at nuclei, below) showed the radius of a gold nucleus to be about 7×10^{-15} m. Most nuclei have radii of a few times **10^{-15} m**, while most atoms have radii of about **10^{-10} m**. So a nucleus is about **10^5 times smaller across** than its atom.

DID YOU KNOW?

If the nucleus were a marble

Make a nucleus the size of a 1 cm marble and put it in the middle of Bhaktapur Durbar Square. The atom, 10^5 times larger, would be a ball about 1 km across, reaching 500 m out on every side, well past Taumadhi Square. The electrons move through all that space; everything else is the marble. Yet the marble holds 99.97 % of the atom's mass.

WORKED EXAMPLE

A 7.7 × 10⁻¹³ J alpha particle

An α-particle travels in a vacuum straight towards the center of a gold nucleus of charge $79e$. Far from the nucleus it has kinetic energy 7.7×10^{-13} J. Treat both as point charges, and take $k = 9.0 \times 10^9$ N m²/C², the mass of the α-particle as 4 u, and $1\text{ u} = 1.66 \times 10^{-27}$ kg.

a. The α-particle does not touch the gold nucleus. Show that the radius of the gold nucleus must be less than 4.7×10^{-14} m.

b. Find the acceleration of the α-particle when the centers of the two are 4.7×10^{-14} m apart.

Answer

a. At the closest approach all the kinetic energy has become potential energy:

$$E_K = \frac{k(79e)(2e)}{d} \Rightarrow d = \frac{9.0 \times 10^9 \times 79 \times 2 \times (1.6 \times 10^{-19})^2}{7.7 \times 10^{-13}} = 4.7 \times 10^{-14}\text{ m}$$

The alpha particle never comes closer than this to the center, and it doesn't touch the nucleus, so the nucleus's radius must be less than 4.7×10^{-14} m.

b. The force at that separation is Coulomb's law:

$$F = \frac{k(79e)(2e)}{r^2} = \frac{9.0 \times 10^9 \times 79 \times 2 \times (1.6 \times 10^{-19})^2}{(4.7 \times 10^{-14})^2} = 16.5\text{ N}$$

$$a = \frac{F}{m} = \frac{16.5}{4 \times 1.66 \times 10^{-27}} = \mathbf{2.5 \times 10^{27}\ m/s^2}$$

That is about 10^{26} times the acceleration of free fall. (The alpha particle's energy here is $7.7 \times 10^{-13} \div 1.6 \times 10^{-13} = 4.8$ MeV.)

Check 2

Doubling the kinetic energy of the alpha particles in a head-on approach to a gold nucleus makes the distance of closest approach:

a twice as large

b half as large

c four times as large

d a quarter as large

24.2 A simple model of the atom

After Rutherford's discovery, the nuclear model was quickly accepted. In 1919 Rutherford found that the nucleus of hydrogen comes out of other nuclei when they are struck by alpha particles; he named it the **proton**. It has a positive charge equal and opposite to the electron's.

But protons alone could not account for an atom's mass. A helium nucleus has twice the charge of a hydrogen nucleus but four times its mass. The puzzle was solved in 1932, when James Chadwick discovered the **neutron**, a particle with no charge and almost the same mass as a proton.

This gives the model of the atom used today:

- **Protons and neutrons** make up the nucleus.
- **Electrons** move around the nucleus in a cloud, some closer to the center and some farther from it. They don't follow fixed paths like planets (see Where is the electron?).

A STORY FROM SCIENCE

The neutron that took twelve years

Rutherford guessed in 1920 that the nucleus must contain a neutral particle, and his student James Chadwick looked for it on and off for twelve years. A neutral particle leaves no track and can't be steered by electric or magnetic fields, so it is almost invisible. In 1932 Irène Joliot-Curie and Frédéric Joliot found that beryllium, struck by alpha particles, gives off a strange radiation that knocks protons out of paraffin wax; they took it for gamma rays. Chadwick read their paper and in less than a month showed that it was a stream of uncharged particles with the mass of a proton. He won the 1935 Nobel Prize in Physics.

42. One of the fusion reactions in a star much more massive than the Sun is the fusion of a calcium nucleus with a helium-4 nucleus to form titanium:

$$^{40}_{20}\text{Ca} + {}^{4}_{2}\text{He} \rightarrow {}^{44}_{22}\text{Ti} + \text{energy}$$

The binding energies per nucleon are: calcium-40, 8.551 MeV; helium-4, 7.073 MeV; titanium-44, 8.533 MeV. Find the energy released in this fusion reaction.

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.

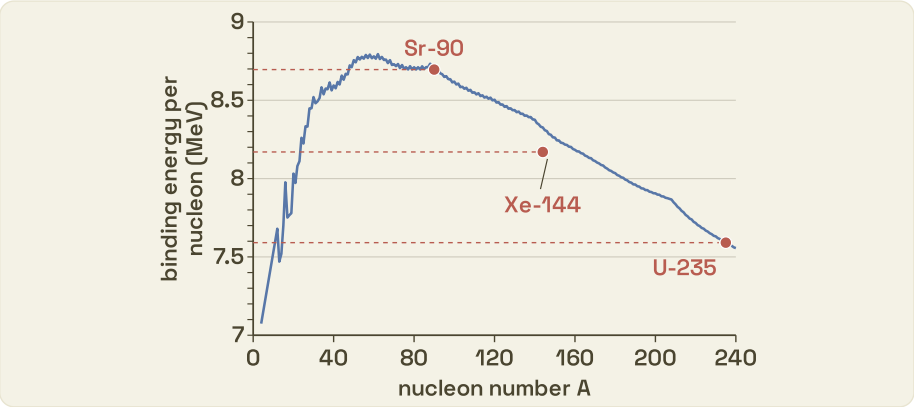


Figure 24.13 *For question 37: the top of the binding energy curve, from 7 to 9 MeV, with uranium-235, strontium-90 and xenon-144 marked. Xenon-144, with many more neutrons than the most stable nuclide of its mass number, lies a little below the curve.*

39. The mass defect of a uranium-235 nucleus is 1.9151 u.
The textbook gives 1.864557 u, which comes from the mass of the whole atom, with its 92 electrons, so the book's numbers don't fit uranium's 7.6 MeV per nucleon; the nucleus's own mass defect is 1.9151 u, as in question 36.
    a. Calculate the binding energy, in joules, of a uranium-235 nucleus. (1 MeV = 1.60×10^{-13} J)
    b. Calculate the average binding energy per nucleon of uranium-235, in MeV.

40. This equation shows a nuclear reaction between two deuterium ($^{2}_{1}\mathrm{H}$) nuclei, forming helium-3:
$$^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}$$
    a. State what type of nuclear reaction this is.
    b. The binding energy per nucleon is 0 for a neutron, about 1.11 MeV for a $^{2}_{1}\mathrm{H}$ nucleus and about 2.58 MeV for a $^{3}_{2}\mathrm{He}$ nucleus. Use these values to estimate the energy released.

41. The Sun radiates about 4×10^{26} W and has a mass of 1.99×10^{30} kg. Astronomers estimate that it can convert only the innermost 10 % of its hydrogen into helium; 73 % of the Sun's mass is hydrogen. Estimate how long the Sun can continue to shine at its present power.

24.3 Some nuclear terminology

Protons and neutrons are together called **nucleons**, because both are found in nuclei. Three numbers describe what a nucleus is made of:

- the **atomic number** or **proton number**, Z: the number of protons;
- the **mass number** or **nucleon number**, A: the total number of nucleons (protons + neutrons);
- the **neutron number**, N: the number of neutrons, so

$$N = A - Z$$

A nucleus with a particular number of protons and neutrons is called a **nuclide**. A nuclide is written

$$^{A}_{Z}\mathrm{X}$$

where X is the chemical symbol of the element. For example, $^{16}_{8}\mathrm{O}$ is an oxygen nucleus with 8 protons and 16 nucleons, so 16 − 8 = 8 neutrons. Since the symbol already fixes Z (every oxygen nucleus has 8 protons), the lower number is often left out: $^{16}\mathrm{O}$, or oxygen-16.

The same notation is used for particles, with the lower number giving the charge (in units of e) and the upper the nucleon number:

Particle	Symbol
proton	$^{1}_{1}\mathrm{p}$ (or $^{1}_{1}\mathrm{H}$)
neutron	$^{1}_{0}\mathrm{n}$
electron	$^{0}_{-1}\mathrm{e}$ (or β^-)
positron	$^{0}_{+1}\mathrm{e}$ (or β^+)
photon	$^{0}_{0}\gamma$, or just γ
alpha particle	$^{4}_{2}\mathrm{He}$ or $^{4}_{2}\alpha$
neutrino	$^{0}_{0}\nu$, or just ν
antineutrino	$^{0}_{0}\bar{\nu}$, or just $\bar{\nu}$

Check 3
The nuclide $^{56}_{26}\mathrm{Fe}$ has:
a 26 neutrons and 56 protons
b 26 protons and 56 neutrons
c 26 protons and 30 neutrons
d 30 protons and 26 neutrons

24.4 Isotopes

Isotopes are nuclides of the same element: they have the **same number of protons** but **different numbers of neutrons**. Ordinary hydrogen ($^{1}_{1}\mathrm{H}$), deuterium ($^{2}_{1}\mathrm{H}$) and tritium ($^{3}_{1}\mathrm{H}$) are all isotopes of hydrogen. An isotope can be named by its element and its mass number: carbon-12 is another way of writing $^{12}_{6}\mathrm{C}$.

The chemistry of an atom is decided by its electrons, and a neutral atom has as many electrons as protons. So all the isotopes of an element, with the same Z, have **almost identical chemical properties**. Their **physical properties** can differ a great deal: one isotope may be strongly radioactive while another is perfectly stable (carbon-14 decays, carbon-12 doesn't). Bombarding a material with neutrons, protons, electrons or other particles can make new, radioactive isotopes; that is how the isotopes used in hospitals are made.

Two more words describe nuclides that share a number:

- **Isotones** have the same **neutron number** N: $^{14}_{6}\mathrm{C}$, $^{15}_{7}\mathrm{N}$, $^{16}_{8}\mathrm{O}$ and $^{17}_{9}\mathrm{F}$ all have 8 neutrons.
- **Isobars** have the same **mass number** A: $^{16}_{6}\mathrm{C}$, $^{16}_{7}\mathrm{N}$, $^{16}_{8}\mathrm{O}$ and $^{16}_{9}\mathrm{F}$ all have 16 nucleons.

A way to keep them apart: isot**o**pes share **p**rotons, isot**o**nes share **n**eutrons, and isob**a**rs share **A**.

Check 4

Which pair are isotopes?

a $^{40}_{18}\text{Ar}$ and $^{40}_{20}\text{Ca}$

b $^{35}_{17}\text{Cl}$ and $^{37}_{17}\text{Cl}$

c $^{14}_{6}\text{C}$ and $^{16}_{8}\text{O}$

d $^{3}_{1}\text{H}$ and $^{3}_{2}\text{He}$

24.5 Atomic mass units

A kilogram suits apples, not protons. For nuclei and particles we use a much smaller unit.

The **atomic mass unit** (u), also called the unified atomic mass unit, is defined as **exactly one twelfth of the mass of an atom of carbon-12**.

Its value in kilograms follows from Avogadro's number. One mole of carbon-12 has a mass of exactly 12 g and contains $N_A = 6.02 \times 10^{23}$ atoms. So one atom has a mass of

$$\frac{12\ \text{g}}{6.02 \times 10^{23}} = \frac{12}{6.02 \times 10^{26}}\ \text{kg} = 12\ \text{u}$$

and

$$1\ \text{u} = \frac{1}{12} \times \frac{12}{6.02 \times 10^{26}}\ \text{kg} = \frac{1}{6.02 \times 10^{26}}\ \text{kg} = \mathbf{1.66 \times 10^{-27}\ kg}$$

On this scale a proton is 1.007 276 u, a neutron 1.008 665 u and an electron 0.000 549 u.

CAREFUL

Atomic mass or nuclear mass?

Tables of "atomic mass" give the mass of the whole **neutral atom**, electrons included. A nucleus is lighter by Z electron masses ($Z \times 0.000\ 549$ u). For uranium-235 that is $92 \times 0.000\ 549 = 0.0505$ u, which matters in a binding-energy calculation. Before you use a mass, check which one it is, and use the proton's mass with a nuclear mass, or the hydrogen atom's mass (1.007 825 u) with an atomic mass.

24.6 Nuclear size and density

The size of a nucleus can be measured by firing very fast electrons at it. Electrons are not affected by the strong nuclear force, so they probe where the charge is; and at high enough energy their de Broglie wavelength (Grade 12) is about 10^{-15} m, small enough to be diffracted by a nucleus. From such experiments, the radius R of a nucleus of mass number A is found to be

$$R = r_0 A^{1/3}, \qquad r_0 \approx 1.2\ \text{fm} \tag{24.1}$$

where 1 fm (one **femtometer**, or **fermi**) $= 10^{-15}$ m. For hydrogen, $A = 1$ and $R = r_0$, so r_0 is roughly the radius of a proton.

The volume of a sphere is $V = \frac{4}{3}\pi R^3$. With equation (24.1),

$$V = \frac{4}{3}\pi \left(r_0 A^{1/3}\right)^3 = \frac{4}{3}\pi r_0^3 A$$

So **the volume of a nucleus is proportional to its number of nucleons**: each nucleon takes up about the same room, like marbles packed in a bag. The mass of the nucleus is about A u, so its density is

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{A\ \text{u}}{\frac{4}{3}\pi r_0^3 A} = \frac{1\ \text{u}}{\frac{4}{3}\pi r_0^3} = \frac{1.66 \times 10^{-27}}{\frac{4}{3}\pi \times (1.2 \times 10^{-15})^3} = \mathbf{2.3 \times 10^{17}\ kg/m^3}$$

A cancels: **all nuclei have about the same density**, whatever their size. This is a key fact about nuclear structure: nucleons pack together like drops of a liquid that can't be squeezed, not like planets round a sun.

A nucleus is small, massive and so extremely dense: about 10^{17} kg/m^3, some 10^{14} times denser than water. A teaspoonful (5 mL) of nuclear matter would have a mass of about $5 \times 10^{-6} \times 2.3 \times 10^{17} \approx 10^{12}$ kg, a thousand million tonnes, about the mass of all the water in Phewa Lake twenty times over.

33. Use these nuclear masses: proton 1.00728 u; neutron 1.00867 u; deuterium 2.01355 u; tritium 3.01550 u; helium-3 3.01493 u; helium-4 4.00151 u.

The textbook gives the proton as 1.00782 u, close to the mass of a whole hydrogen atom, and only three reactions, which add up to about 39 MeV, so the book's numbers don't fit; with the proton's own mass, 1.00728 u, and the fourth reaction in part d, they do.

a. Two deuterium nuclei fuse: $^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \to {}^{3}_{2}\text{He} + {}^{1}_{0}\text{n}$.

1. Show that the change in mass in this reaction is about 3.5×10^{-3} u.
2. Determine the energy in MeV that is given out.

b. Deuterium and tritium fuse to give a helium-4 nucleus and a neutron: $^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \to {}^{4}_{2}\text{He} + {}^{1}_{0}\text{n}$.

1. Show that the change in mass in this reaction is about 1.9×10^{-2} u.
2. Determine the energy in MeV that is given out.

c. Deuterium and helium-3 react to form helium-4 and a proton: $^{2}_{1}\text{H} + {}^{3}_{2}\text{He} \to {}^{4}_{2}\text{He} + {}^{1}_{1}\text{p}$.

1. Show that the change in mass is about 0.02 u.
2. Determine the energy in MeV.

d. Two deuterium nuclei can also fuse the other way: $^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \to {}^{3}_{1}\text{H} + {}^{1}_{1}\text{p}$. Together, these four reactions turn six deuterium nuclei into two helium-4 nuclei, two protons and two neutrons.

1. Show that the total energy released by the four reactions is about 43 MeV.
2. This set of reactions happens 6.0×10^{23} times. Calculate the total energy released in J.
3. Calculate the total mass that has been turned into energy.

34. In a certain fission reaction of uranium-235, the mass change is 3.1×10^{-28} kg for each nucleus.

1. Determine the energy equivalent to this mass change.
2. Calculate how many nuclei would be needed to give 1.0 J. Give your answer to an appropriate number of significant figures.

35. A nucleus written in isotope notation is $^{A}_{Z}\text{X}$. It has a mass M. The mass of a proton is m_p and the mass of a neutron is m_n.

1. Write an expression for the mass defect Δm of the nucleus.
2. Write an expression for its binding energy per nucleon, E.

36. a. Uranium-235 has the nuclide notation $^{235}_{92}\text{U}$. Use these data to calculate its binding energy per nucleon: mass of uranium-235 nucleus = 234.993 u; proton = 1.00728 u; neutron = 1.00867 u.

The textbook gives 235.044 u, which is the mass of the whole atom, with its 92 electrons; with bare protons and neutrons that number gives 7.39 MeV per nucleon instead of uranium's 7.6, so the book's numbers don't fit. The nucleus alone is 234.993 u.

b. Figure 24.12 shows how the binding energy per nucleon varies with the nucleon number A.

1. State where uranium-235 and its fission fragments are on the graph. (No values are needed.)
2. Explain why fission releases energy.

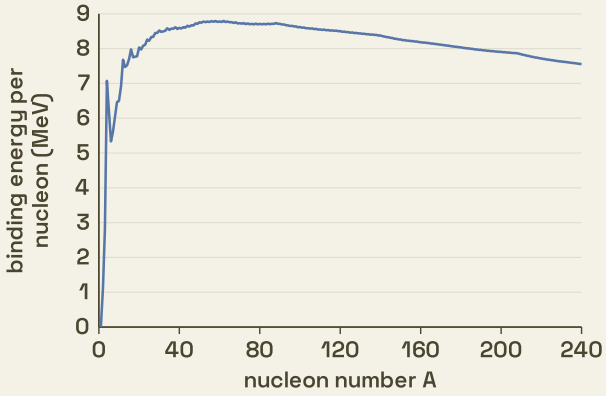


Figure 24.12 *For question 36b: the binding energy per nucleon against the nucleon number A.*

Check 17

24. The mass defect of a carbon-12 nucleus is 0.0990 u. What is its average binding energy per nucleon?

a 8.01 MeV b 1.19 MeV
c 92.2 MeV d 7.68 MeV

Answer these questions

25. The most common isotope of calcium is $^{40}_{20}$Ca. Its nuclear mass is 39.9516 u.
The textbook gives 39.9626 u, which is the mass of the whole atom, with its 20 electrons; the nucleus alone is 39.9516 u, so the book's numbers don't fit with bare protons and neutrons.
   a. Calculate the mass defect of this nucleus.
   b. Calculate its binding energy in MeV.

   (Mass of a proton = 1.00728 u; mass of a neutron = 1.00867 u.)

26.
   a. Show that the mass defect of a helium nucleus (alpha particle) is 0.03039 u.
   b. Calculate its binding energy per nucleon in MeV.

   (Mass of a helium nucleus = 4.00151 u; proton = 1.00728 u; neutron = 1.00867 u.)

27.
   a. Write down an equation to show how helium is formed from hydrogen by fusion.
   b. Explain the conditions needed for the reaction to occur.

28. Calculate the mass of material that is converted into 5.0×10^{13} J of energy.

29. A typical alpha decay is the decay of radon-220:

$$^{220}_{86}\text{Rn} \rightarrow {}^{A}_{Z}\text{X} + \alpha + \text{energy}$$

   a. Give the values of A and Z for both X and α.
   b. Calculate the energy released, in MeV and in J, by one nucleus of radon. (Atomic masses: radon-220, 220.011 394 u; polonium-216, 216.001 915 u; helium-4, 4.002 603 u.)

   The textbook gives no masses for part b, so the book's numbers don't work as asked; the atomic masses here are from the tables (the electrons balance, 86 on each side).

30. In this reaction, a proton and a deuterium nucleus fuse to form a helium nucleus, releasing energy:

$$^{2}_{1}\text{H} + {}^{1}_{1}\text{p} \rightarrow {}^{A}_{2}\text{He} + \text{energy}$$

   a. Write down the isotope of helium formed, in isotope notation.
   b. Calculate the energy given out by this reaction. (Mass of proton = 1.00728 u; deuterium nucleus = 2.01355 u; helium nucleus = 3.01493 u.)

31. Beryllium-7 decays by **electron capture** to lithium: its nucleus captures one of the atom's own inner electrons.

$$^{7}_{4}\text{Be} + {}^{0}_{-1}\text{e} \rightarrow {}^{7}_{3}\text{Li} + \nu_e$$

   Calculate the energy change in this event. (Mass of beryllium-7 nucleus = 7.014 727 u; lithium-7 nucleus = 7.014 356 u; electron = 0.000 549 u; the neutrino's mass is negligible.)

32. a. Beryllium-9 has a proton number of 4. Its nuclear mass is 9.00999 u.
   1. Calculate the number of neutrons beryllium-9 has.
   2. Calculate the mass defect of the beryllium-9 nucleus.

   b. Beryllium-8 has a nuclear mass of 8.00311 u.
   1. Calculate the mass defect of a beryllium-8 nucleus.
   2. Show that the energy equivalent to the mass defect of a beryllium-8 nucleus is about 56.5 MeV.
   3. Calculate its binding energy per nucleon.

   c. The binding energy of each helium-4 nucleus, worked out from its mass defect, is 28.31 MeV. Two alpha particles fuse to form beryllium-8: $^{4}_{2}\text{He} + {}^{4}_{2}\text{He} \rightarrow {}^{8}_{4}\text{Be}$.
   1. Calculate the total binding energy of the two alpha particles.
   2. Calculate the energy change as the reaction proceeds.

   (Mass of proton = 1.00728 u; neutron = 1.00867 u.)
   The textbook gives 9.01218 u and 8.00531 u, which are the masses of the whole atoms, with their 4 electrons each; with bare protons and neutrons those numbers don't work out right (they give 54.48 MeV for beryllium-8, and a reaction in c that needs 2 MeV, ten times too much). The nuclei alone are 9.00999 u and 8.00311 u.

WORKED EXAMPLE

The radius of a gold nucleus

Find the radius of a gold-197 nucleus, and compare it with the 5.7×10^{-14} m that the 4.0 MeV alpha particles reach.

Answer

$$R = r_0 A^{1/3} = 1.2 \times 10^{-15} \times 197^{1/3} = 1.2 \times 10^{-15} \times 5.82 = \mathbf{7.0 \times 10^{-15}\ m}$$

The alpha particles stop about 8 times farther out than the nuclear surface, so they never touch it: Rutherford's distance of closest approach was a safe upper limit.

DID YOU KNOW?

Stars made of nuclear matter

A **neutron star** is the collapsed core of a large star, left after it explodes as a supernova. Its matter is squeezed to about the density of a nucleus, so a neutron star with 1.4 times the Sun's mass is only about 20 km across, about the length of the Kathmandu Valley. It is, in effect, one giant nucleus held together by gravity.

YOUR TURN

How big is uranium-238?

Find the radius of a uranium-238 nucleus, and how many times larger it is across than a helium-4 nucleus.

Answer

$R = 1.2 \times 10^{-15} \times 238^{1/3} = 1.2 \times 10^{-15} \times 6.20 = \mathbf{7.4 \times 10^{-15}\ m}$.
$\frac{R_\text{U}}{R_\text{He}} = \left(\frac{238}{4}\right)^{1/3} = 59.5^{1/3} = \mathbf{3.9}$. Uranium has nearly 60 times as many nucleons, but is less than 4 times as wide: volume grows as A, radius only as A^⅓.

24.7 Einstein's mass and energy

In 1905, working on special relativity, Albert Einstein showed that mass and energy are two forms of the same thing, linked by

$$E = mc^2$$

where E is energy in joules, m is mass in kilograms, and c is the speed of light, 3.00×10^8 m/s. Because c^2 is so large, 9×10^{16} m²/s², a tiny mass is worth an enormous energy. The first experiment to show it directly was done in 1932, when John Cockcroft and Ernest Walton split lithium nuclei with fast protons and found that the energy of the fragments matched the mass that had disappeared.

Where the mass goes when a nucleus forms

A helium nucleus contains four nucleons, two protons and two neutrons. Its mass is slightly **smaller** than the mass of the four nucleons apart:

- mass of a proton = 1.6726×10^{-27} kg
- mass of a neutron = 1.6749×10^{-27} kg
- mass of two protons and two neutrons = $2 \times 1.6726 \times 10^{-27} + 2 \times 1.6749 \times 10^{-27} = 6.6950 \times 10^{-27}$ kg
- measured mass of a helium nucleus = 6.6447×10^{-27} kg
- difference = $6.6950 \times 10^{-27} - 6.6447 \times 10^{-27} = 5.03 \times 10^{-29}$ kg

When the four nucleons come together to form a helium nucleus, this mass is given out as energy:

$$E = mc^2 = 5.03 \times 10^{-29} \times (3.00 \times 10^8)^2 = 4.53 \times 10^{-12}\ \text{J} = \frac{4.53 \times 10^{-12}}{1.60 \times 10^{-13}}\ \text{MeV} = 28.3\ \text{MeV}$$

That is about the energy of seven million chemical bonds, from one helium nucleus. In Einstein's own words, "very small amounts of mass may be converted into a very large amount of energy and vice versa".

The reverse is also true. To pull a helium nucleus apart into its four nucleons, work must be done against the strong forces that hold them together, 28.3 MeV of it, and that energy put in reappears as the extra mass of the separate nucleons.

Mass defect and binding energy

A nucleus of Z protons and N neutrons always has **less mass** than its protons and neutrons apart. The difference is called its **mass defect**, Δm:

$$\Delta m = Z m_p + N m_n - M_{\text{nucleus}} \tag{24.2}$$

measured in kilograms or in atomic mass units. The energy that corresponds to it is the **binding energy** of the nucleus:

$$\text{binding energy} = \Delta m \times c^2 \tag{24.3}$$

The binding energy is the energy that would have to be supplied to a nucleus to pull it apart into separate protons and neutrons, and it is the energy given out when the nucleus is formed from them. It is measured in J or, more often, in MeV.

CAREFUL

Binding energy is not energy stored in the nucleus

A nucleus with a large binding energy does not "contain" that energy: it has **less** energy (and less mass) than its separate nucleons, because that energy has already been given out. Binding energy is what you would have to **put in** to take it apart. The more tightly bound a nucleus, the larger its binding energy and the lower its mass.

24.7.1 Mass–energy equivalence

The equation $E = mc^2$ can be read two ways.

Mass is a form of energy. When an electron meets its antiparticle, a positron, they destroy each other (**annihilation**) and their entire mass becomes the energy of two gamma-ray photons (section 24.8).

Energy has mass. Whenever a system gains or loses energy ΔE, its mass changes by Δm, where

$$\Delta E = \Delta m\, c^2 \tag{24.4}$$

A cup of hot tea loses a little mass as it cools; a stretched spring has a little more mass than the same spring slack; a moving ball has a little more energy, and so more inertia, than the same ball at rest. In everyday life the changes are far too small to notice, because c^2 is so large:

- A sealed torch that gives out 10 W of light for 10 hours (36 000 s) loses $10 \times 36\,000 = 0.36$ MJ of energy. Its mass falls by $\dfrac{3.6 \times 10^5}{(3.0 \times 10^8)^2} = 4.0 \times 10^{-12}$ kg, nothing compared with the mass of the torch.
- A 1000 kg car that speeds up from rest to 30 m/s gains $\frac{1}{2} \times 1000 \times 30^2 = 450$ kJ of kinetic energy, worth $\dfrac{4.5 \times 10^5}{9.0 \times 10^{16}} = 5.0 \times 10^{-12}$ kg.
- An unstable nucleus that gives out a 5 MeV gamma-ray photon loses $5 \times 1.6 \times 10^{-13} = 8.0 \times 10^{-13}$ J. Its mass falls by $\dfrac{8.0 \times 10^{-13}}{9.0 \times 10^{16}} = 8.9 \times 10^{-30}$ kg: about **0.5 %** of the mass of a nucleon, far from negligible on a nucleus's scale.

Electrons in a particle accelerator can be given energies hundreds of times their rest energy, and there the extra inertia is very noticeable: it is why no accelerator can push an electron to the speed of light.

WORKED EXAMPLE

A person in a moving car

A person of mass 70 kg sits in a car that is at rest, then moves at a steady 15 m/s. By how much has the person's mass increased?

Answer

The gain in energy is the kinetic energy, $\Delta E = \frac{1}{2}mv^2$:

$$\Delta m = \frac{\Delta E}{c^2} = \frac{\frac{1}{2} \times 70 \times 15^2}{(3.00 \times 10^8)^2} = \frac{7875}{9.00 \times 10^{16}} = \mathbf{8.8 \times 10^{-14}\ kg} \approx 10^{-13}\ \text{kg}$$

A minute change, far beyond any scale's power to notice: a single speck of dust, 10 micrometers across, has about ten times that mass.

The energy of one atomic mass unit

In nuclear calculations it is handy to know the energy equivalent of 1 u:

$$E = (1\ \text{u})\,c^2 = (1.660539 \times 10^{-27})(2.997925 \times 10^8)^2 = 1.492418 \times 10^{-10}\ \text{J}$$

and, since 1 eV = 1.602177×10^{-19} J,

Check 15

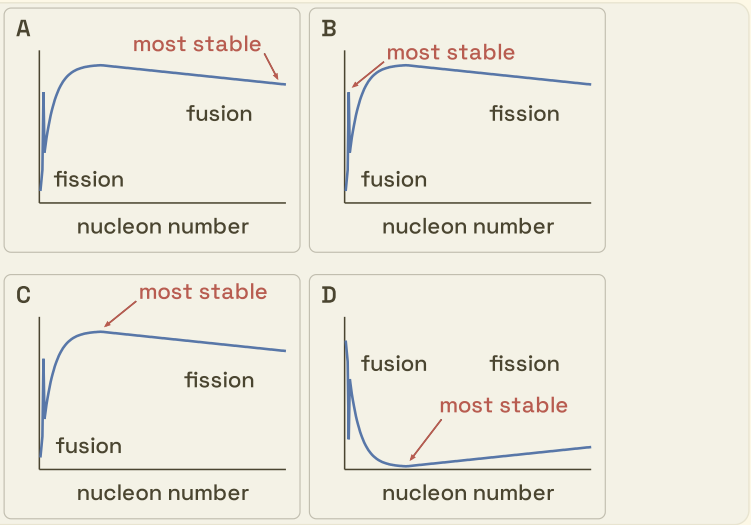


Figure 24.10 *For question 22: four sketches of the binding energy per nucleon against nucleon number, each labeling where fission and fusion happen and the most stable nuclide.*

22. Which one of the graphs of binding energy per nucleon against nucleon number above is correct?

a A
b B
c C
d D

Check 16

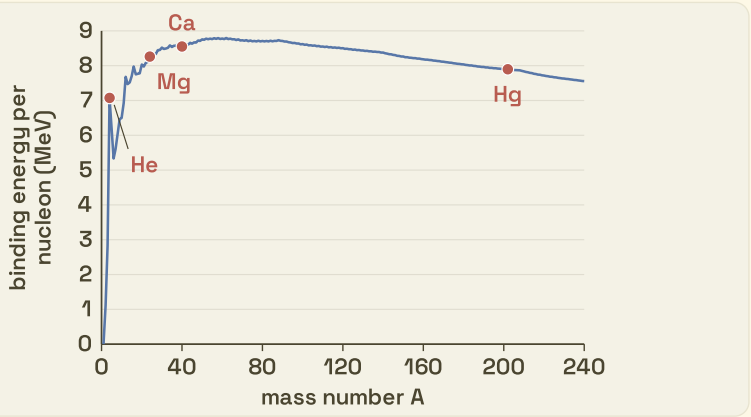


Figure 24.11 *For question 23: binding energy per nucleon against mass number, with helium-4, magnesium-24, calcium-40 and mercury-202 marked.*

23. From the graph above, which of the four nuclei is the most stable?

a helium (He)
b magnesium (Mg)
c calcium (Ca)
d mercury (Hg)

Check 12

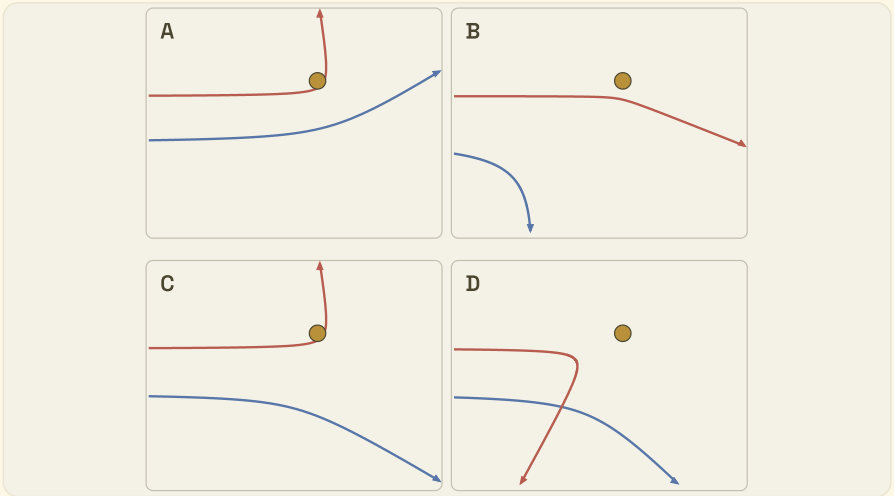


Figure 24.9 *For question 19: two alpha particles with the same kinetic energy move towards a gold nucleus (the gold dot), both passing below it, the upper (red) one aimed closer to it. Four suggestions for their paths, A to D.*

19. Two alpha particles with the same kinetic energy are moving towards, and are then deflected by, a gold nucleus. Which diagram above could show the paths of the two alpha particles?

a A
b B
c C
d D

Check 13

20. In an experiment to investigate the nature of the atom, a very thin gold film was bombarded with α-particles. What pattern of deflection of the α-particles was observed?

a A few α-particles were deflected through angles greater than a right angle.
b All α-particles were deflected from their original path.
c Most α-particles were deflected through angles greater than a right angle.
d No α-particle was deflected through an angle greater than a right angle.

Check 14

21. The numbers of protons, neutrons and nucleons in three nuclei are: X, 15 protons, 16 neutrons, 31 nucleons; Y, 15 protons, 17 neutrons, 32 nucleons; Z, 16 protons, 16 neutrons, 32 nucleons. Which nuclei are isotopes of the same element?

a X and Y
b X and Z
c Y and Z
d none of them

$$E = \frac{1.492418 \times 10^{-10}}{1.602177 \times 10^{-19}}\ \text{eV} = 931.494 \times 10^{6}\ \text{eV}$$

So **1 u is equivalent to 931.5 MeV**. A mass defect in u, multiplied by 931.5, gives the binding energy in MeV straight away.

Nuclear reactions turn mass into energy and energy into mass, but nothing is lost: **in any closed system, the total of the energy and the energy equivalent of the mass is constant.** This is the law of conservation of energy, with mass counted as one of the forms of energy.

Check 5

A system gives out 9.0×10^{13} J of energy. Its mass decreases by:

a 1 kg
b 1 g
c 1 mg
d 9×10^{13} kg

24.8 Annihilation and creation

When a particle meets its own **antiparticle** they destroy each other: **annihilation**. The positron is the electron's antiparticle, with the same mass and the opposite charge (Unit 26). When an electron and a positron meet, their whole mass becomes energy, in the form of **two gamma-ray photons** that fly off in opposite directions (two, not one, so that momentum is conserved).

The smallest energy of the photons, for a pair that meets nearly at rest:

- change in mass: $\Delta m = 2m_e$, where $m_e = 9.1 \times 10^{-31}$ kg is the mass of an electron or a positron;
- energy released: $\Delta E = \Delta m c^2 = 2m_e c^2$, shared by two equal photons;
- energy of each photon: $m_e c^2 = 9.1 \times 10^{-31} \times (3.00 \times 10^{8})^2 = 8.2 \times 10^{-14}$ J, or **0.51 MeV**.

If the electron and positron were also moving, each photon has even more energy.

This is not science fiction: hospitals use it. In **positron emission tomography (PET)**, a patient is given a sugar-like substance tagged with a radioactive isotope (often fluorine-18) that gives out positrons. Each positron soon meets an electron in the body and annihilates; a ring of detectors round the patient catches the two 0.51 MeV photons arriving at the same instant from opposite sides, and a computer works out where they came from. The result is a map of where the body uses the most sugar, which shows active parts of the brain, or a tumor.

The opposite of annihilation is **pair production**: a single photon disappears, and its energy creates a particle and its antiparticle. This happens when a high-energy gamma-ray photon passes close to a heavy nucleus (the nucleus is needed to take up some momentum). To make an electron–positron pair, the photon must supply at least the rest energy of both particles:

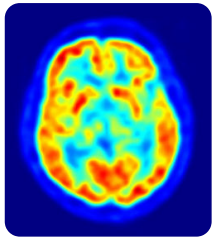


Figure 24.4 *A PET scan of the brain, one slice seen from above. Red and yellow mark where the most tagged sugar was taken up, and so where the most positrons annihilated: the busiest parts of the brain.*

Photo: Jens Maus (http://jens-maus.de/), Public domain

$$E_{\text{min}} = 2m_e c^2 = 2 \times 0.51\ \text{MeV} = \mathbf{1.02\ MeV}\ (1.64 \times 10^{-13}\ \text{J})$$

Any energy above that becomes the kinetic energy of the pair.

Check 6

A gamma-ray photon of 0.80 MeV passes close to a lead nucleus. Can it produce an electron–positron pair?

a yes, because 0.80 MeV is more than the electron's rest energy of 0.51 MeV
b no, because it needs at least 1.02 MeV to make both particles
c yes, if the lead nucleus supplies the rest of the energy
d no, because photons can never produce matter

24.9 Forces in the nucleus

Apart from hydrogen, every nucleus contains two or more protons, packed together within a few femtometers. Like charges repel, and at that distance the repulsion is enormous. What keeps nuclei from flying apart? Could it be gravity?

WORKED EXAMPLE

Can gravity hold two protons together?

Compare the gravitational attraction and the electrostatic repulsion between two protons in a nucleus. (Proton mass $m = 1.67 \times 10^{-27}$ kg, charge $q = 1.60 \times 10^{-19}$ C, $G = 6.67 \times 10^{-11}$ N m²/kg², $k = 8.99 \times 10^{9}$ N m²/C².)

Answer

The gravitational force is $F_g = \dfrac{Gm_1m_2}{r^2}$; the electrostatic force is $F_e = \dfrac{kq_1q_2}{r^2}$. Their ratio is

$$\frac{F_g}{F_e} = \frac{Gm_1m_2/r^2}{kq_1q_2/r^2} = \frac{Gm_1m_2}{kq_1q_2}$$

The r^2 cancels, so the answer is the same at every distance. Substituting,

$$\frac{F_g}{F_e} = \frac{(6.67 \times 10^{-11})(1.67 \times 10^{-27})^2}{(8.99 \times 10^{9})(1.60 \times 10^{-19})^2} = \mathbf{8.08 \times 10^{-37}}$$

For gravity to hold the protons together this ratio would have to be more than 1. It is about 10^{36} times too small. **Gravity cannot be what holds a nucleus together.**

Since gravity is far too weak, there must be another force. It is called the **strong nuclear force** (or strong force), and it is one of the four fundamental forces of nature, with gravity, the electromagnetic force and the weak nuclear force.

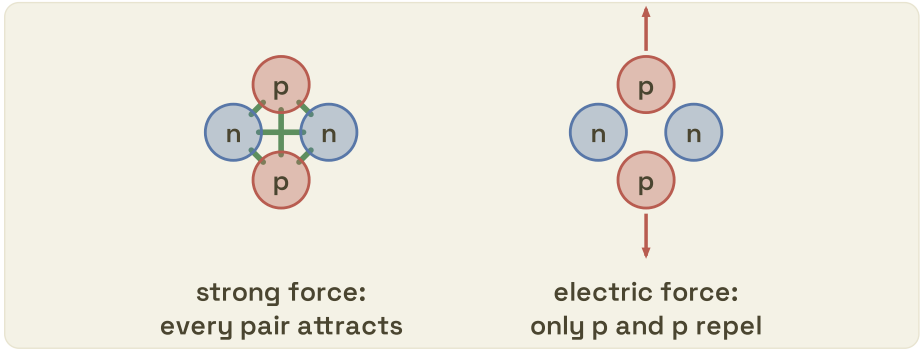


Figure 24.5 *In a helium nucleus every nucleon attracts every neighbor by the strong nuclear force (left), protons and neutrons alike. Only the two protons repel each other, by the electric force (right). At nuclear distances the strong force wins.*

- It is **attractive** between nucleons and acts on **protons and neutrons alike**; it does not act on electrons.
- It is **very strong**: within a nucleus, stronger than the electric repulsion between protons.
- It has a **very short range**, about 10^{-15} m (1 fm). Beyond a few femtometers it falls almost to nothing. At separations below about 0.5 fm it becomes **repulsive**, which is why nucleons don't collapse into each other and why nuclei all have the same density.

Neutrons are the glue. They add strong attraction with no electric repulsion, which is why heavy nuclei need more neutrons than protons: lead-208 has 82 protons and 126 neutrons.

Check 7

Which statement about the strong nuclear force is correct?

a It acts between protons only, and repels them.
b It has an infinite range, like gravity.
c It attracts protons and neutrons alike, but only over about 10^{-15} m.
d It holds electrons in their orbits round the nucleus.

IN SHORT

Key ideas of Unit 24

- Rutherford's alpha-scattering experiment: most alpha particles pass straight through a thin gold foil, about 1 in 8000 is turned back through more than 90°. The atom is mostly empty, with its positive charge and nearly all its mass in a tiny nucleus.
- Distance of closest approach: $E_K = \dfrac{k(2e)(Ze)}{d}$; it is an upper limit for the nuclear radius. Nuclei are about 10^{-15} m across, atoms about 10^{-10} m.
- ${}^{A}_{Z}\mathrm{X}$: Z protons, A nucleons, $N = A - Z$ neutrons. Isotopes: same Z; isotones: same N; isobars: same A.
- 1 u = $\frac{1}{12}$ of the mass of a carbon-12 atom = 1.66×10^{-27} kg ≡ 931.5 MeV.
- $R = r_0A^{1/3}$, $r_0 \approx 1.2$ fm; all nuclei have a density of about 2.3×10^{17} kg/m³.
- $E = mc^2$, $\Delta E = \Delta m\,c^2$. Mass defect $\Delta m = Zm_p + Nm_n - m_{\text{nucleus}}$; binding energy = $\Delta m\,c^2$, the energy needed to separate a nucleus into its nucleons.
- Electron–positron annihilation gives two photons of at least 0.51 MeV each; pair production needs a photon of at least 1.02 MeV.
- The strong nuclear force: attractive between all nucleons, stronger than the electric force in a nucleus, range about 1 fm. Gravity is 10^{36} times too weak.
- Binding energy per nucleon: zero for ¹H, peaks at about 8.8 MeV round iron-56 and nickel-62, 7.6 MeV at uranium; 7–9 MeV for most nuclei. Fusion of light nuclei and fission of heavy ones both move towards the peak and release energy.
- Conservation of charge, nucleon number and mass–energy in nuclear reactions; energy released $\Delta E = (m_i - m_f)c^2$.
- Fission: ${}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}\mathrm{n}$, about 173 MeV; a chain reaction is steady at $k = 1$ (reactor), explosive at $k > 1$ (bomb).
- Fusion needs millions of kelvin. The proton–proton chain turns 4 ¹H into ⁴He + $2e^+ + 2\nu + 2\gamma$, giving 26.7 MeV.

Exercise

Choose the correct option

Check 11

18. In the early 20th century, experiments were carried out to measure the scattering of alpha particles after striking thin gold foil. Which of the following could **not** be concluded from the results of these experiments?

a The nucleus contains most of the mass of the atom.
b The nucleus is made of protons and neutrons.
c The nucleus is charged.
d The nucleus has a much smaller radius than the radius of the atom.

WORKED EXAMPLE

How long can the Sun shine?

The Sun radiates about 4×10^{26} W and has a mass of 1.99×10^{30} kg, 73 % of it hydrogen. Astronomers estimate that it can convert only the hydrogen in its innermost 10 % into helium. Estimate how long the Sun can continue to shine at its present power.

Answer

Each helium nucleus formed gives out 26.71 MeV $= 26.71 \times 1.60 \times 10^{-13}$ J $= 4.27 \times 10^{-12}$ J. The rate at which helium is produced:

$$\frac{\text{power of the Sun}}{\text{energy per helium nucleus}} = \frac{4 \times 10^{26}\ \text{J/s}}{4.27 \times 10^{-12}\ \text{J}} = 9.36 \times 10^{37}\ \text{per second}$$

Each helium nucleus uses 4 hydrogen atoms, each of mass 1.67×10^{-27} kg, so hydrogen is used at

$$4 \times 9.36 \times 10^{37} \times 1.67 \times 10^{-27} = 6.25 \times 10^{11}\ \text{kg/s}$$

about 600 million tonnes every second. The hydrogen available is 10 % of 73 % of the Sun's mass:

$$t = \frac{1.99 \times 10^{30} \times 0.73 \times 0.10}{6.25 \times 10^{11}} = 2.32 \times 10^{17}\ \text{s} = \frac{2.32 \times 10^{17}}{3.16 \times 10^{7}}\ \text{years} \approx \mathbf{7 \times 10^{9}\ years}$$

The Sun can shine at its present rate for about **7 billion years** in all. It is about 4.6 billion years old, so it is a little past halfway.

DID YOU KNOW?

Fusion on Earth

Fusion power would run on deuterium from seawater and lithium, leaving far less long-lived radioactive waste than fission, which is why countries have spent seventy years trying to tame it. The difficulty is keeping a gas at over 100 million kelvin, hotter than the Sun's core, away from every wall. In December 2022 the National Ignition Facility in California, using 192 lasers to crush a peppercorn-sized capsule of deuterium and tritium, got more fusion energy out (3.15 MJ) than the laser energy put in (2.05 MJ), for the first time. The ITER reactor, being built in France by 35 countries, is planned to hold a burning plasma with magnets.

Check 10

Why does fusion need temperatures of millions of kelvin?

a so that the nuclei melt

b to break the nuclei into protons and neutrons first

c so that the nuclei move fast enough to overcome their electric repulsion and get within range of the strong force

d because the strong force only acts at high temperatures

24.10 Binding energy

The strong nuclear force acts only between **neighboring** nucleons, over about 10^{-15} m, and the very existence of stable nuclei with many protons in them is evidence for it.

To take a nucleus completely apart, work must be done against the strong force to pull every proton and neutron away from the others. This work is the **binding energy** of the nucleus: **the energy needed to separate all its protons and neutrons and move them infinitely far apart.**

Run the process backwards and build a nucleus from separate nucleons: as the strong force pulls them together, energy is **given out**, exactly as much as the binding energy. So energy is needed to take a nucleus apart, and the same energy is released when it is put together, just as energy is needed to pull a ball back up a hill it rolled down.

24.11 Mass defect

Deuterium is the isotope of hydrogen whose nucleus, the **deuteron**, is one proton bound to one neutron. To separate them, work must be done against the strong force. By $E = mc^2$, the separated proton and neutron must then have **more mass** than the deuteron.

Is this really true? Masses can be measured very accurately with a mass spectrometer:

- mass of a deuteron = 2.013 553 u
- mass of a proton = 1.007 276 u
- mass of a neutron = 1.008 665 u
- proton + neutron = 2.015 941 u

The separated nucleons are indeed heavier, by

$$\Delta m = 2.015\,941 - 2.013\,553 = 0.002\,388\ \text{u}$$

which is equivalent to 0.002 388 × 931.5 = **2.2 MeV** (3.6×10^{-13} J). So at least 2.2 MeV is needed to break a deuteron into a proton and a neutron; and when a proton and a neutron join to form a deuteron, 2.2 MeV is given out, usually as a gamma-ray photon. Both have been measured, and agree with the masses.

The difference 0.002 388 u is the **mass defect** of the deuteron, and 2.2 MeV is its **binding energy**:

- The **mass defect** of a nucleus is the difference between the total mass of its separated nucleons and the mass of the nucleus. In other words, the mass of a nucleus is less than the total mass of its protons and neutrons.
- The **binding energy** of a nucleus is the minimum energy needed to separate it completely into its protons and neutrons. It is the mass defect × c^2.

24.11.1 Calculating a binding energy

For a nuclide ${}^{A}_{Z}\mathrm{X}$ with $N = A - Z$ neutrons and nuclear mass m_{nucleus},

$$\Delta m = Z m_p + N m_n - m_{\text{nucleus}}$$

$$\text{binding energy} = \Delta m\, c^2 = \left(Z m_p + N m_n - m_{\text{nucleus}}\right) c^2$$

With masses in u, the binding energy in MeV is $\Delta m \times 931.5$.

A uranium-235 nucleus has 92 protons and 143 neutrons. To split it into all its nucleons takes far more than 2.2 MeV: there are many more nucleons, and many more neighbors holding each one.

WORKED EXAMPLE

The binding energy of uranium-235

Find the binding energy of a $^{235}_{92}\mathrm{U}$ nucleus. Nuclear mass of uranium-235 = 234.993 422 u; proton mass = 1.007 276 u; neutron mass = 1.008 665 u; 1 u = 1.66×10^{-27} kg.

Answer

The nucleus has 92 protons and $235 - 92 = 143$ neutrons.

Mass of the separated nucleons = $92 \times 1.007\,276 + 143 \times 1.008\,665 = 92.669\,392 + 144.239\,095 = 236.908\,487$ u.

Mass defect: $\Delta m = 236.908\,487 - 234.993\,422 = 1.915\,065$ u $= 1.915\,065 \times 1.66 \times 10^{-27} = 3.18 \times 10^{-27}$ kg.

Binding energy = $\Delta m\, c^2 = 3.18 \times 10^{-27} \times (3.0 \times 10^{8})^2 = \mathbf{2.86 \times 10^{-10}\ J}$.

In MeV: $\dfrac{2.86 \times 10^{-10}}{1.60 \times 10^{-13}} = \mathbf{1790\ MeV}$ (with 1 u = 931.5 MeV, $1.915\,065 \times 931.5 = 1784$ MeV; the small difference comes from rounding c and the u).

Per nucleon, $1784 \div 235 = \mathbf{7.6\ MeV}$.

CAREFUL

Use the nuclear mass with bare protons

Uranium-235's **atomic** mass is 235.043 930 u, which includes its 92 electrons. Subtracting it from bare protons and neutrons gives a mass defect too small by $92 \times 0.000\,549 = 0.0505$ u, about 47 MeV, and a binding energy per nucleon of 7.39 MeV instead of 7.59 MeV. Use nuclear mass with m_{p}, or atomic mass with the hydrogen atom's mass (1.007 825 u), which puts back the same number of electrons on each side.

24.11.2 The binding energy per nucleon

Bigger nuclei have bigger binding energies, simply because they have more nucleons. To compare how **tightly** different nuclei are held, divide by the number of nucleons:

$$\text{binding energy per nucleon} = \frac{\text{binding energy}}{A}$$

For deuterium it is 2.2 MeV ÷ 2 = **1.1 MeV**; for uranium-235, 7.6 MeV. The binding energy per nucleon is the **average energy needed to remove one nucleon** from the nucleus. The greater it is, the more tightly bound the nucleus, and the more **stable** it is.

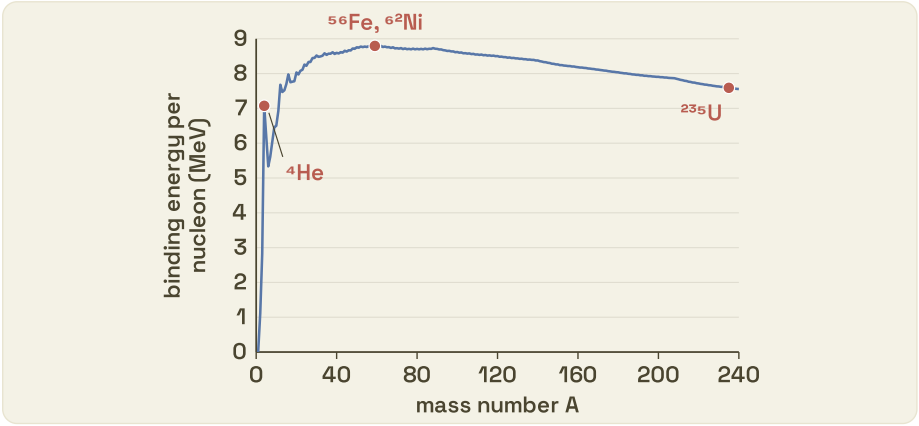


Figure 24.6 *The binding energy per nucleon of the nuclides, against the mass number A (from measured masses). From $A = 9$ on, each point is the most tightly bound nuclide of that mass number. The curve rises steeply through the light nuclei, peaks at about 8.8 MeV round iron and nickel, and then falls slowly to 7.6 MeV at uranium.*

The shape of this graph explains radioactive decay, fission and fusion. From it you can see that:

- The binding energy per nucleon of ordinary hydrogen, $^{1}_{1}\mathrm{H}$, is **zero**: its nucleus is a single proton, with nothing to bind.
- The curve **rises sharply** for small A: for nuclei up to $A \approx 56$, the binding energy per nucleon increases as A increases.

the Sun waits about ten billion years for it.

Next, the deuteron fuses with another proton, forming helium-3 and a gamma-ray photon, and giving out **5.5 MeV**:

$$^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{p} \to {}^{3}_{2}\mathrm{He} + \gamma$$

Last, two helium-3 nuclei (each made by the first two steps) fuse to form helium-4 and two protons, giving out **12.9 MeV**:

$$^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \to {}^{4}_{2}\mathrm{He} + 2\,{}^{1}_{1}\mathrm{p}$$

The two protons given back can start the cycle again. The first two steps happen twice for each third step, so overall

$$4\,{}^{1}_{1}\mathrm{H} \to {}^{4}_{2}\mathrm{He} + 2\,{}^{0}_{+1}\mathrm{e} + 2\nu + 2\gamma$$

and the total energy is $2 \times 1.44 + 2 \times 5.5 + 12.9 = \mathbf{26.7\ MeV}$. This chain, also called **hydrogen burning**, is one of the main ways helium is made in stars. In the Sun it happens about 9×10^{37} times every second.

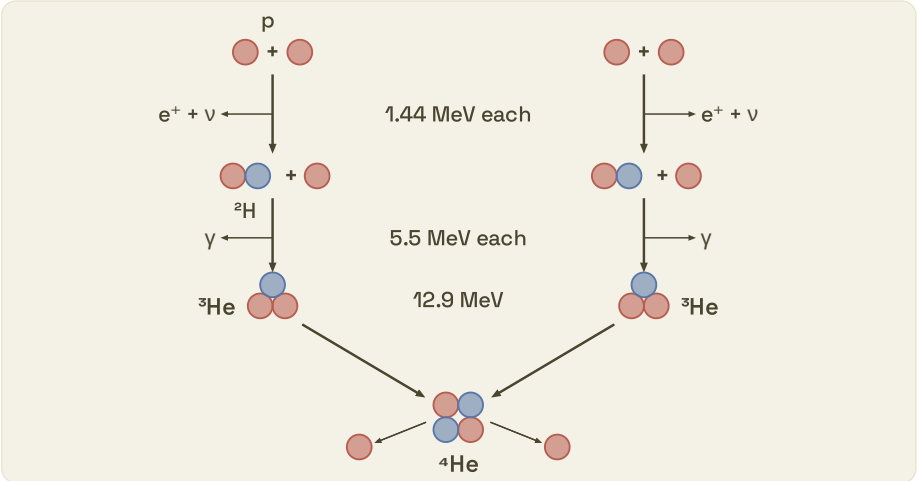


Figure 24.8 *The proton–proton chain. Twice, two protons fuse to form a deuteron (giving out a positron and a neutrino), and the deuteron fuses with a third proton to form helium-3 (giving out a gamma ray). The two helium-3 nuclei then fuse into helium-4, giving back two protons.*

CAREFUL

The first step does not give out 2.2 MeV

It is tempting to say that two protons forming a deuteron give out the deuteron's binding energy, 2.2 MeV. But one proton must turn into a neutron, which is heavier, and a positron must be created: those take 1.29 MeV and 0.51 MeV. What is left is $2.22 - 1.29 - 0.51 = \mathbf{0.42\ MeV}$, and 1.44 MeV once the positron has annihilated. Only with these values do the three steps add up to the 26.7 MeV of the whole chain.

WORKED EXAMPLE

The energy of the proton–proton chain

Find the total energy released when four protons become one helium-4 nucleus. Use atomic masses: hydrogen-1, 1.007 825 u; helium-4, 4.002 603 u (with atomic masses, the electrons that the two positrons annihilate are already counted).

Answer

Initial mass: $4 \times 1.007\,825 = 4.031\,300$ u.

Change in mass: $4.031\,300 - 4.002\,603 = 0.028\,697$ u.

Energy released: $0.028\,697$ u $\times$ 931.494 MeV/u = **26.7 MeV**.

This is 0.7 % of the mass, shared among the helium nucleus, the gamma rays and the neutrinos (which carry off about 2 % of it, straight out of the Sun).

What happens depends on the average number of neutrons from each fission that go on to cause another fission, the **multiplication factor** k. The rest escape from the fuel or are absorbed without causing fission.

- $k < 1$: each generation has fewer fissions than the last, and the chain dies out (**subcritical**).
- $k = 1$: each fission leads to exactly one more; the rate stays steady (**critical**). A nuclear reactor runs here, controlled by rods that absorb spare neutrons.
- $k > 1$: the number of fissions grows by a factor k every generation (**supercritical**). A generation takes only about 10 nanoseconds in a bomb, so with $k = 2$ the whole energy is released in less than a microsecond.

A chain reaction can thus be made to go slowly and steadily, as in a **nuclear reactor**, or explosively, as in a **bomb**. Either way the energy released is enormous, far greater than in any chemical reaction: the fission of 1 kg of uranium-235 gives about 8×10^{13} J, as much as burning about 3000 tonnes of coal.

DID YOU KNOW?

A natural nuclear reactor, two billion years ago

In 1972, French scientists checking uranium ore from Oklo, in Gabon, found that it contained slightly too little uranium-235. The missing atoms had been split: about two billion years ago, when natural uranium was richer in uranium-235 than it is now, groundwater seeping into the ore slowed its neutrons enough for chain reactions to run by themselves. Sixteen natural reactors ran on and off for hundreds of thousands of years, switching off whenever the heat boiled the water away. They are the only natural fission reactors known.

A STORY FROM SCIENCE

The first chain reaction

On 2 December 1942, under the stands of a football field at the University of Chicago, Enrico Fermi's team pulled the last control rod out of "Chicago Pile-1", a stack of about 350 tonnes of graphite blocks with about 50 tonnes of uranium and uranium oxide set in them. The neutron counters clicked faster and faster, and stayed fast: k was just above 1. Fermi let it run for 4.5 minutes, at half a watt, then had the rods put back. It was the first controlled nuclear chain reaction, and the first nuclear reactor.

Check 9

In a nuclear reactor working steadily, the multiplication factor k is:

a 0
b less than 1
c 1
d 3, the number of neutrons from each fission

24.14 Nuclear fusion

Nuclear fusion is the building up of nuclei by bringing protons and neutrons together, or by joining small nuclei into larger ones.

Joining small nuclei gives out energy, because the total binding energy of the fused nucleus is larger than the total binding energy of the nuclei that formed it. The difference is given out as the kinetic energy of the products.

The catch is getting the nuclei to touch. The strong force only reaches about 10^{-15} m, so two nuclei must come within a few femtometers of each other. But all nuclei are positive, and at that distance their electric repulsion is enormous. At ordinary temperatures nuclei never get close enough to fuse. At very high temperatures they move fast enough to get close despite the repulsion, and the strong force pulls them together. That is why fusion is also called a **thermonuclear** reaction.

The conditions are right in the **cores of stars**. In the Sun's core the temperature is about 1.5×10^7 K and the density about 1.5×10^5 kg/m^3 (150 times the density of water); the enormous density makes the number of fusions per second high.

The proton–proton chain

Fusion in stars happens in chains of reactions. In the Sun, most of the energy comes from the **proton–proton chain**:

First, two protons fuse to form a deuteron, a positron and a neutrino:

$$ {}_1^1\mathrm{p} + {}_1^1\mathrm{p} \rightarrow {}_1^2\mathrm{H} + {}_{+1}^{0}\mathrm{e} + \nu $$

This step gives out 0.42 MeV, and the positron soon annihilates with an electron, adding 1.02 MeV more: **1.44 MeV** in all. One of the protons has turned into a neutron, through the weak nuclear force, which is why this step is very slow: an average proton in

- It has **peaks** at ${}_2^4\mathrm{He}$, ${}_6^{12}\mathrm{C}$ and ${}_8^{16}\mathrm{O}$, which are unusually stable compared with their neighbors. All three are made of whole alpha particles (2, 3 and 4 of them), and helium-4 is so tightly bound that heavy nuclei often throw out an alpha particle when they decay.
- The **highest** binding energy per nucleon, about **8.8 MeV**, is round **iron-56 and nickel-62**: these are the most tightly bound nuclei in nature.
- Beyond the peak, the curve **falls gently**: for A above about 60, the binding energy per nucleon decreases as A increases, to 7.6 MeV at uranium. In large nuclei every proton repels every other proton, near or far, while the strong force only holds neighbors, so the repulsion gradually wins.
- **Most nuclei** have a binding energy per nucleon **between 7 and 9 MeV**, nearly the same. That is the short range of the strong force at work: a nucleon is held only by its immediate neighbors, not by all the nucleons in the nucleus. Once a nucleus is big enough (roughly $A > 20$), each nucleon inside has about the same number of neighbors, so it takes about the same energy to remove, whatever the size of the nucleus.

CAREFUL

Iron-56 or nickel-62?

Many books say iron-56 has the greatest binding energy per nucleon. By a hair, nickel-62 has (8.795 MeV against 8.790 MeV for iron-56, with iron-58 in between). Iron-56 is the nucleus with the smallest **mass per nucleon**, and it is far more common, because it is the end of the chain of fusion in heavy stars. In an exam, "iron-56 (and nickel-62), at the peak, about 8.8 MeV" is right either way.

What the graph says about energy:

- **Fusion.** When two light nuclei (left of the peak) join to make a heavier one, the product is higher up the curve: it has a larger binding energy per nucleon, and more total binding energy than the nuclei that formed it. The difference is given out. Fusion is how the Sun and the other stars produce their energy; thanks to it, there is life on Earth.
- **Fission.** When a heavy nucleus (far right) splits into two middle-sized ones, the fragments are also higher up the curve, and again the difference in binding energy is given out. All nuclear power stations work this way.
- **Radioactive decay.** In a spontaneous decay, such as a heavy nucleus giving out an alpha particle, the total binding energy of the daughter nucleus and the alpha particle is greater than that of the parent nucleus. The difference appears as the kinetic energy of the products.

Both fusion and fission move nuclei **towards the peak**. Nuclei at the peak can give out energy neither way: iron is the ash of the nuclear fire.

Check 8

Why do both fusion of light nuclei and fission of heavy nuclei give out energy?

a both reactions destroy protons
b in both, the products have a greater binding energy per nucleon than what went in
c in both, the products have a smaller binding energy per nucleon than what went in
d both reactions add neutrons from outside

24.12 Nuclear reactions

Some nuclei can give out energy by fission or fusion. Almost all nuclear reactions that happen naturally produce nuclei that are more stable, with a higher binding energy per nucleon than the original nuclei. The **mass** of the products is then less than the mass of the original nuclei, and the difference in mass corresponds to the energy released. To increase their binding energy per nucleon, **light nuclei tend to fuse**, and **heavy nuclei tend to undergo radioactive decay or fission**.

WORKED EXAMPLE

Energy from the binding energy curve

Reading the binding energies per nucleon from Figure 24.6: deuterium 1.11 MeV, tritium 2.83 MeV, helium-4 7.07 MeV; uranium-235 7.59 MeV, rubidium-92 8.51 MeV, cesium-140 8.31 MeV.

a. Estimate the energy released when a deuterium and a tritium nucleus fuse to form helium-4 and a neutron.

b. Estimate the energy released when a uranium-235 nucleus splits into rubidium-92, cesium-140 and a few neutrons.

Answer

The energy released is the **total** binding energy of the products minus that of what went in (a neutron alone has none). Total binding energy = binding energy per nucleon × A.

a. Before: deuterium $2 \times 1.11 = 2.22$ MeV; tritium $3 \times 2.83 = 8.49$ MeV; in all **10.7 MeV**.
After: helium-4, $4 \times 7.07 =$ **28.3 MeV**.
Energy released $\approx 28.3 - 10.7 =$ **17.6 MeV**.

b. Before: uranium-235, $235 \times 7.59 =$ **1784 MeV**.
After: rubidium-92, $92 \times 8.51 = 783$ MeV; cesium-140, $140 \times 8.31 = 1163$ MeV; in all **1946 MeV**.
Energy released $\approx 1946 - 1784 \approx$ **160 MeV**.

Per nucleon of fuel, fusion gives $17.6 \div 5 = 3.5$ MeV, fission only $160 \div 236 = 0.7$ MeV: fusion is about five times richer, kilogram for kilogram.

Conservation laws in nuclear reactions

Besides momentum and energy, every nuclear reaction obeys these conservation laws:

- **Charge (proton number).** The total charge cannot change. Any change in the charge of a nucleus is exactly balanced by an opposite change elsewhere: if a nucleus loses $+2e$ of charge, a particle of charge $+2e$ (an alpha particle) must be given out. In nuclide symbols, the **lower numbers add up to the same total** on both sides.
- **Nucleon number (mass number).** The total number of nucleons stays the same: the **upper numbers add up to the same total** on both sides.
- **Mass–energy.** The total of the mass (as energy, mc^2) and all other energy stays the same.

For example, in ${}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}\mathrm{n}$: the upper numbers give $1 + 235 = 236 = 141 + 92 + 3$, and the lower numbers $0 + 92 = 92 = 56 + 36 + 0$.

YOUR TURN

Find the missing nucleus

Complete: ${}^{14}_{7}\mathrm{N} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{17}_{8}\mathrm{O} + ?$. (This is the reaction Rutherford found in 1919, the first change of one element into another made by people.)

Answer

Upper numbers: $14 + 4 = 17 + A$, so $A = 1$. Lower numbers: $7 + 2 = 8 + Z$, so $Z = 1$. The missing particle is ${}^{1}_{1}\mathrm{H}$, **a proton**: the way the proton was discovered.

24.13 Nuclear fission

Nuclear fission is the splitting of a heavy, unstable nucleus into **two fragments of comparable mass**, usually with a few neutrons.

On the binding energy curve, a heavy nucleus is far to the right. Split it into two nuclei near the middle, and the binding energy per nucleon **increases**: the energy difference is given out, mostly as the kinetic energy of the fission fragments. Any nucleus with A above about 120 would release energy by splitting, though in practice only a few, uranium and plutonium above all, can be made to split easily.

For fission and fusion alike, the energy released is

$$\Delta E = E_{b,f} - E_{b,i} = (\text{increase in the total mass defect}) \times c^2$$

where $E_{b,i}$ is the total binding energy of the original nucleus or nuclei and $E_{b,f}$ that of the products. Since binding energies correspond to mass defects, and the numbers of protons and neutrons don't change, the increase in the total mass defect is exactly the **decrease in the total mass**. So the energy released corresponds to the mass the reaction turns into energy:

$$\Delta E = (m_i - m_f)\,c^2 \tag{24.5}$$

where m_i is the total mass of the original nucleus or nuclei and m_f the total mass of the products. Energy is released when $m_f < m_i$.

Fission usually begins with a **free neutron** striking a heavy nucleus. A neutron has no charge, so it is not repelled and can drift right into a nucleus even when moving slowly. When uranium-235 absorbs a neutron, it becomes uranium-236 for a moment, a nucleus so disturbed that it wobbles, stretches into a dumbbell and splits almost at once. One possible result:

$${}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{236}_{92}\mathrm{U} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}\mathrm{n}$$

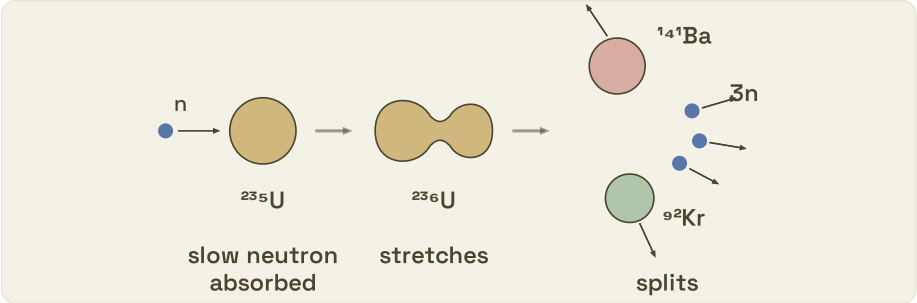


Figure 24.7 *A slow neutron is absorbed by a uranium-235 nucleus. The uranium-236 nucleus formed stretches and splits into two fragments, here barium-141 and krypton-92, which fly apart, and three fast neutrons.*

WORKED EXAMPLE

The energy of one fission

Find the energy released in the fission above. Masses: uranium-235, 235.043 930 u; neutron, 1.008 665 u; barium-141, 140.914 412 u; krypton-92, 91.926 156 u. (These are atomic masses: the electrons balance, 92 on each side.)

Answer

Total mass before:
$m_i = m_{\mathrm{U}} + m_n = 235.043\,930 + 1.008\,665 = 236.052\,595$ u
Total mass after:
$m_f = m_{\mathrm{Ba}} + m_{\mathrm{Kr}} + 3m_n = 140.914\,412 + 91.926\,156 + 3(1.008\,665) = 235.866\,563$ u
Mass lost:
$m_i - m_f = 236.052\,595 - 235.866\,563 = 0.186\,032$ u
Energy released, with 1 u = 931.5 MeV:
$\Delta E = 0.186\,032 \times 931.5 =$ **173.3 MeV**
That is about 0.08 % of the mass. Burning one carbon atom gives about 4 eV, so one fission gives some 40 million times more.

Where does the energy of one fission go? About 205 MeV in all (nuclear-data tables differ by a few MeV), shared like this:

Where the energy goes	About
kinetic energy of the two fragments, which heats the fuel as they crash into the atoms around them	168 MeV
kinetic energy of the 2 or 3 neutrons	5 MeV
gamma rays given out at the moment of splitting	7 MeV
beta particles and gamma rays the fragments give out later, as they decay	15 MeV
antineutrinos, which pass out of any reactor and are lost	10 MeV

More than four-fifths of it is the fragments' kinetic energy. These are averages over the many ways uranium-235 splits. The 173 MeV worked out above is for one split, into barium-141 and krypton-92, and counts only the moment of splitting; the fragments' later decays, and the antineutrinos they send out, add some 25 MeV more.

The chain reaction

Each fission is set off by one neutron, and gives out two or three more. If at least one of these goes on to split another uranium-235 nucleus, the fissions keep going by themselves: a **chain reaction**. The fission is **induced**: it starts when a neutron strikes a uranium-235 nucleus.